

Worksheet 1: Limits - Solutions

1. $\lim_{x \rightarrow 2} (3x^4 - 2x^2 + x - 1) = 3(2)^4 - 2(2)^2 + 2 - 1 = 3(16) - 2(4) + 1 = 41$

2. $\lim_{x \rightarrow 4} \sqrt{16 - x^2} = \sqrt{16 - 2^2} = 0$

3. $\lim_{x \rightarrow 4} \frac{x^2 - 4x}{x^2 - 3x - 4} \rightarrow \frac{16 - 16}{16 - 12 - 4} \rightarrow \frac{0}{0} \rightarrow ?$

$$\frac{x^2 - 4x}{x^2 - 3x - 4} = \frac{x(x - 4)}{(x + 1)(x - 4)} = \frac{x}{x + 1} \rightarrow \frac{4}{4 + 1} = \frac{4}{5}$$

4. $\lim_{x \rightarrow 1} \frac{x^3 - 1}{x^2 - 1} \rightarrow \frac{1 - 1}{1 - 1} \rightarrow \frac{0}{0} \rightarrow ?$

$$\frac{x^3 - 1}{x^2 - 1} = \frac{(x - 1)(x^2 + x + 1)}{(x - 1)(x + 1)} = \frac{x^2 + x + 1}{x + 1} \rightarrow \frac{1 + 1 + 1}{1 + 1} = \frac{3}{2}$$

5. $\lim_{x \rightarrow -4} \frac{\sqrt{x^2 + 9} - 5}{x + 4} \rightarrow \frac{0}{0} \rightarrow ?$

$$\begin{aligned} \frac{\sqrt{x^2 + 9} - 5}{x + 4} &= \frac{\sqrt{x^2 + 9} - 5}{x + 4} \times \frac{\sqrt{x^2 + 9} + 5}{\sqrt{x^2 + 9} + 5} = \frac{(\sqrt{x^2 + 9})^2 - (5)^2}{(x + 4)(\sqrt{x^2 + 9} + 5)} \\ &= \frac{x^2 + 9 - 25}{(x + 4)(\sqrt{x^2 + 9} + 5)} = \frac{x^2 - 16}{(x + 4)(\sqrt{x^2 + 9} + 5)} \\ &= \frac{(x - 4)(x + 4)}{(x + 4)(\sqrt{x^2 + 9} + 5)} = \frac{x - 4}{\sqrt{x^2 + 9} + 5} \rightarrow \frac{-4 - 4}{\sqrt{16 + 9} + 5} = -\frac{8}{10} = -\frac{4}{5} \end{aligned}$$

6. $\lim_{h \rightarrow 0} \frac{(h - 1)^3 + 1}{h} \rightarrow \frac{-1 + 1}{0} \rightarrow \frac{0}{0} \rightarrow ? =$

$$\frac{(h - 1)^3 + 1}{h} = \frac{h^3 - 3h^2 + 3h - 1 + 1}{h} = \frac{h^3 - 3h^2 + 3h}{h} = h^2 - 3h + 3 \rightarrow 3$$

7. Find $\lim_{h \rightarrow 0} \frac{f(a + h) - f(a)}{h}$ for $f(x) = x^2$ with $a = 2$.

$$\frac{f(a + h) - f(a)}{h} = \frac{(2 + h)^2 - 2^2}{h} = \frac{4 + 4h + h^2 - 4}{h} = \frac{4h + h^2}{h} = 4 + h \rightarrow 4$$

8. Find $\lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$ for $f(x) = \sqrt{3x + 1}$ with $a = 3$.

$$\frac{f(x) - f(a)}{x - a} = \frac{\sqrt{3x + 1} - \sqrt{3(3) + 1}}{x - 3} = \frac{\sqrt{3x + 1} - \sqrt{10}}{x - 3}$$

$$\begin{aligned} \frac{\sqrt{3x + 1} - \sqrt{10}}{x - 3} &= \frac{\sqrt{3x + 1} - \sqrt{10}}{x - 3} \times \frac{\sqrt{3x + 1} + \sqrt{10}}{\sqrt{3x + 1} + \sqrt{10}} = \frac{3x + 1 - 10}{(x - 3)(\sqrt{3x + 1} + \sqrt{10})} \\ &= \frac{3(x - 3)}{(x - 3)(\sqrt{3x + 1} + \sqrt{10})} = \frac{3}{\sqrt{3x + 1} + \sqrt{10}} \rightarrow \frac{3}{2\sqrt{10}} \end{aligned}$$