Math 103 Section 3.1, 3.2:

Math of Finance: solving for time

Three ways to compute future value

Simple interest	$A = P(1 + rt)$
Compound interest	$A = P(1 + i)^n$
Continuous compounded interest	$A = Pe^{rt}$

These formulas can also be used to compute the present value required to attain a given future value.

Example: What present value P is required for a future value F of \$4,000? Interest is compounded semiannually for 5 years at a rate of 8%.

Solve the equation for P:

$$\begin{aligned} 4000 &= P(1 + .08/2)^{10} \\ &= P(1.48024) \\ P &= 4000/1.48024 \\ &= 2702.26 \end{aligned}$$

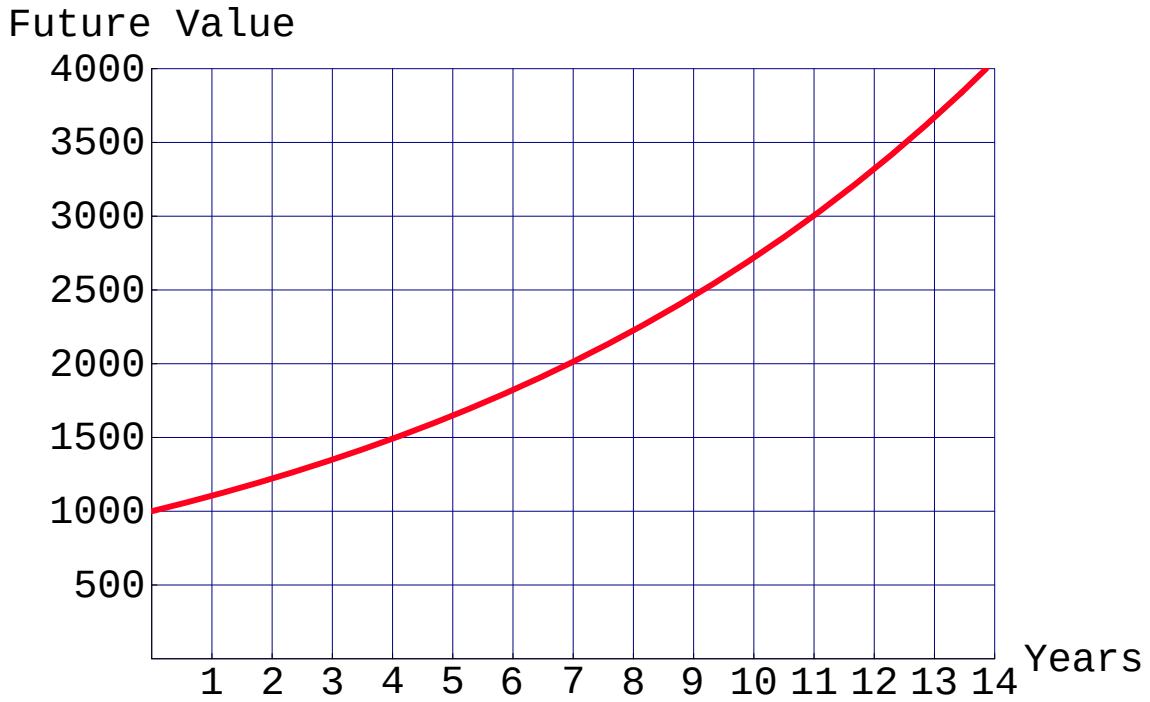
Summary: A present value of \$2702.26 is required for a future value of \$4000 if interest is compounded semiannually for 5 years at a rate of 8%.

Solving the future/present value formula for time t

Example: A present value of \$1000 is invested at 10% compounded continuously. How many years are required for a future value of \$3000?

Use the graph to solve the equation for the number of years t:

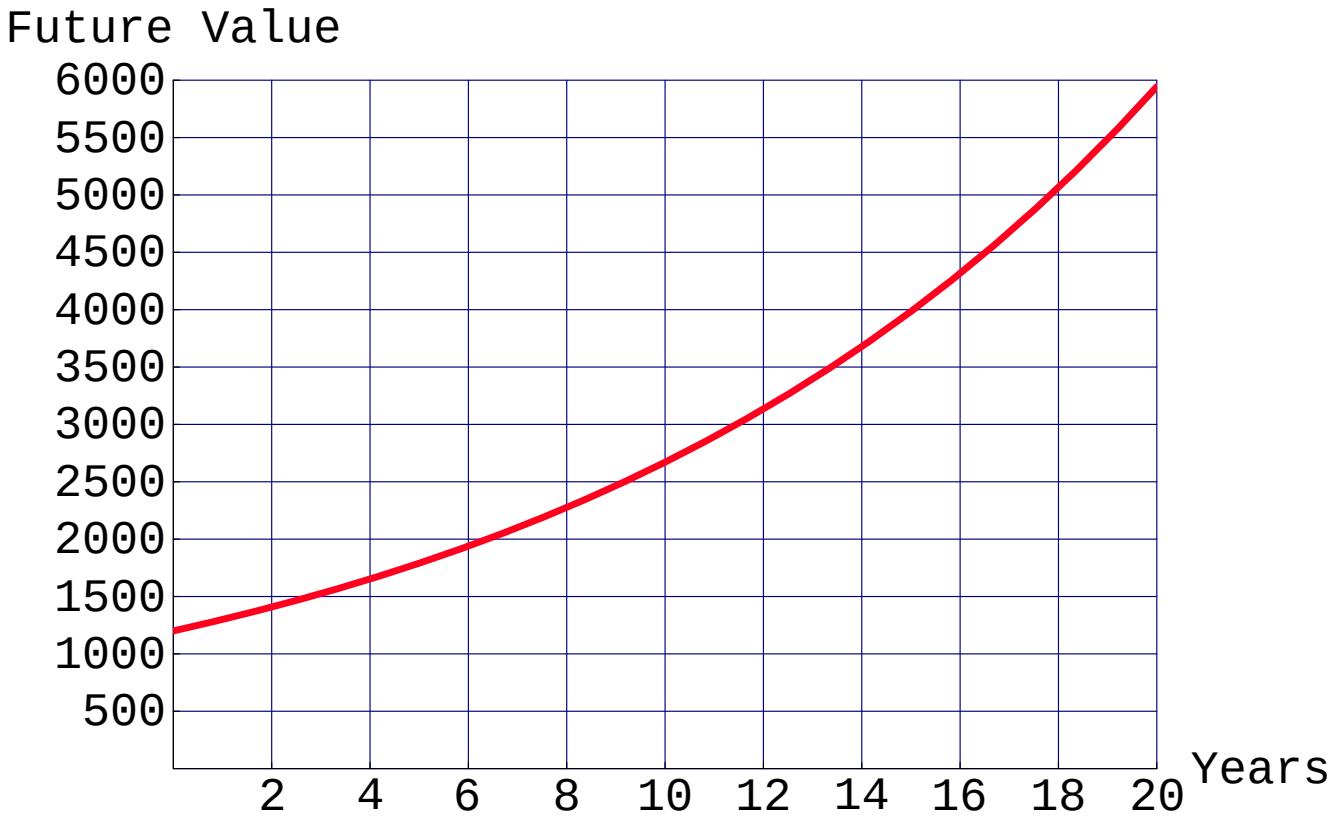
$$3000 = 1000e^{(.10)t}$$



Solving the future/present value formula for time t

Use logarithms graph to solve the equation for the number of years t:

$$5000 = 1200e^{(.08)t}$$



Two facts about the natural logarithm, ln:

$$\ln\left(e^{x}\right) = x \tag{1}$$

$$\ln\left(a^{x}\right) = x\ln(a) \tag{2}$$

Fact (1):		Fact (2):	
$\ln\left(e^{(.03)t}\right)$	$= (.03)t$	$\ln\left((1.02)^{n}\right)$	$= n\ln(1.02)$
			$= (0.0198026)n$
$\ln\left(e^{(.09)t}\right)$	$= (.09)t$	$\ln\left((1.10)^{n}\right)$	$= n\ln(1.10)$
			$= (0.0953102)n$
$\ln\left(e^{(.06)t}\right)$	$=$	$\ln\left((1.045)^{n}\right)$	$= n\ln(1.045)$
			$= (0.0440169)n$
$\ln\left(e^{(.10)t}\right)$	$= (.10)t$	$\ln\left((1.01)^{n}\right)$	$= n\ln(1.01)$
			$=$

Solving the future/present value formula for time t

Use logarithms graph to solve the equation for the number of years t:

$$3000 = 1000e^{(.10)t}$$

$$\begin{aligned} e^{(.10)t} &= 3 \\ \ln\left(e^{(.10)t}\right) &= \ln(3) \\ (.10)t &= \ln(3) \\ t &= \ln(3)/.10 \\ &= 10.98612 \end{aligned}$$

Summary: It takes 10.98612 years for a present value of \$1000 to grow to a future value of \$3000 at a rate of 10% compounded continuously

Solving the future/present value formula for time t

Use logarithms graph to solve the equation for the number of years t:

$$2500 = 1000e^{(.09)t}$$

$$3600 = 1000e^{(.05)t}$$

How to use Fact (2):

Compound Interest Formula: $A = P(1+i)^n$

Problem: Deposit \$100 into an account earning 4.5% interest compounded annually. How many years will it take to have a future value of \$200? Solve for n:

$$200 = 100(1+.045)^n.$$

$$\begin{aligned} (1.045)^n &= 2 \\ \ln\left((1.045)^n\right) &= \ln(2) \\ n\ln(1.045) &= \ln(2) \\ n &= \ln(2)/\ln(1.045) \\ n = &= 15.747302 \end{aligned}$$

Summary: It takes 15.75 years to have a future value of \$200 if a present value of \$100 earns 4.5% interest compounded annually.

How to use Fact (2):

Compound Interest Formula: $A = P(1 + i)^n$

Problem: Deposit \$100 into an account earning 9% interest compounded semiannually. How many years will it take to have a future value of \$200? Solve for n:

$$200 = 100(1 + .045)^n.$$

This is the same equation as in the previous slide. The answer is still $n = 15.747302$, but it must be interpreted differently. n is the number of compounding periods. n isn't always the number of years. In this example, interest is compounded semiannually. So 15.747302 periods is $15.747302/2 = 7.873651$ years.

Summary: It takes 7.87 years to have a future value of \$200 if a present value of \$100 earns 9% interest compounded semiannually.

Example: The present value is \$200. Interest is compounded quarterly at a rate of 10%. How many years does it take for a future value of \$500?

Compound interest formula: $A = P(1+i)^n$
$i = 0.10/4 = 0.025$ Solve for n:

$$\begin{aligned} 200(1.025)^n &= 500 \\ (1.025)^n &= 2.5 \\ \ln\left((1.025)^n\right) &= \ln(2.5) \\ n\ln(1.025) &= \ln(2.5) \\ n &= \ln(2.5)/\ln(1.025) \\ n &= 37.107890 \end{aligned}$$

There are 37.108 periods. Each period is a quarter (of a year). So that's $37.108/4 = 9.277$ years.

Summary: It takes 9.277 years to have a future value of \$500 if a present value of \$200 earns 10% compounded quarterly.

Problem: The present value is $1200. Interest is compounded monthly at a rate of 8%. How many years does it take for a future value of $2000?

Problem: The present value is $1800. Interest is compounded quarterly at a rate of 12%. How many years does it take for a future value of $3200?