WARM UP EXERCSE

A company makes "Notebook" computers. The price-demand function is

$$p(x) = 1{,}190 - 36x.$$

Write the company's revenue function and graph it on a domain of $0 < x < 35$.

§ 2-2 Elementary Functions: Graphs and Transformations

The student will learn about:

- **A library of elementary functions**
- **Their graphs**
- **Shifts and stretches of these**

A Beginning Library of Elementary Functions

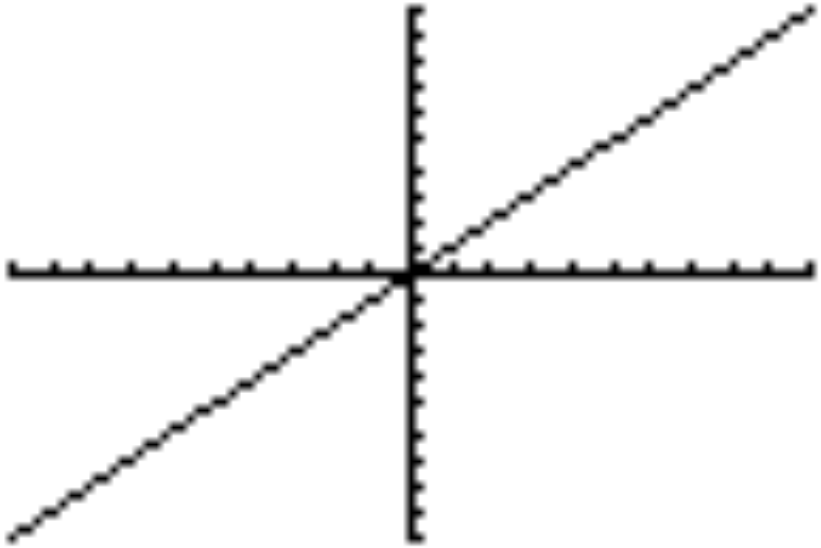
The identity function; $f(x) = x$**:**

Domain:

Range:

$f(2)=$

$f(3.456)=$



A Beginning Library of Elementary Functions

The square function; $h(x) = x^2$

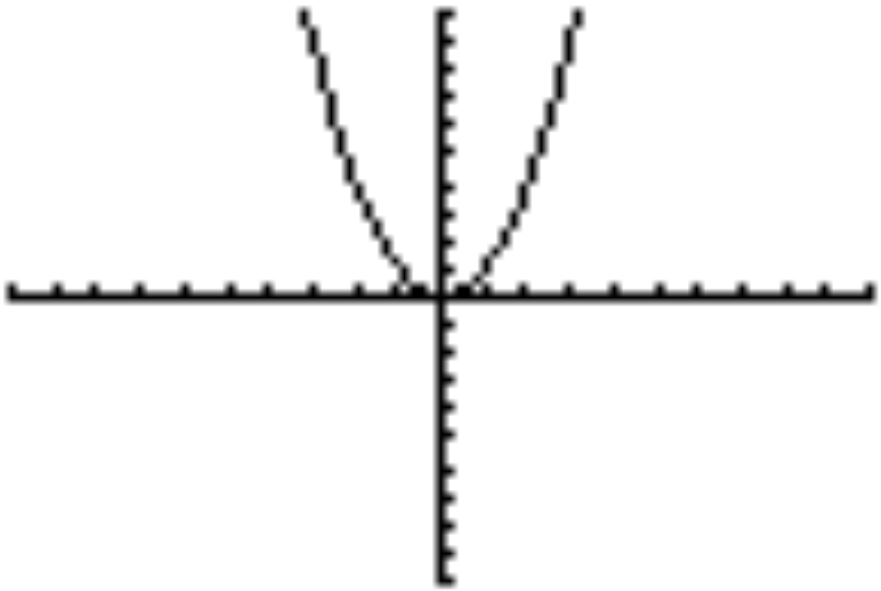
Domain:

Range:

$h(0)=$

$h(2)=$

$h(100)=$



A Beginning Library of Elementary Functions

The cube function; $m(x) = x^3$

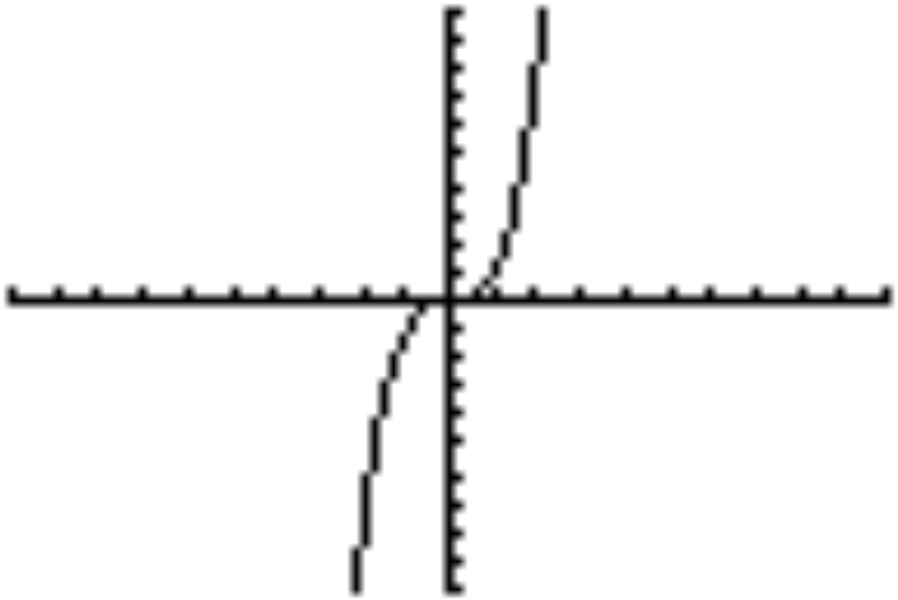
Domain:

Range:

m(0)=

m(2)=

m(100)=



A Beginning Library of Elementary Functions

The cube function; $n(x) = \sqrt{x}$

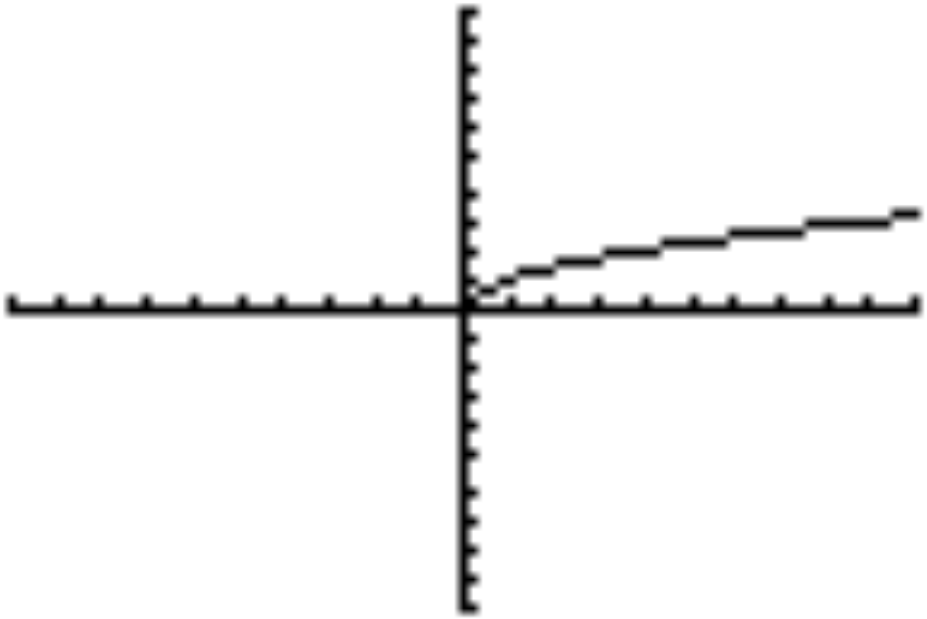
Domain:

Range:

n(0)=

n(2)=

n(100)=



A Beginning Library of Elementary Functions

The cube function; $p(x) = \sqrt[3]{x}$

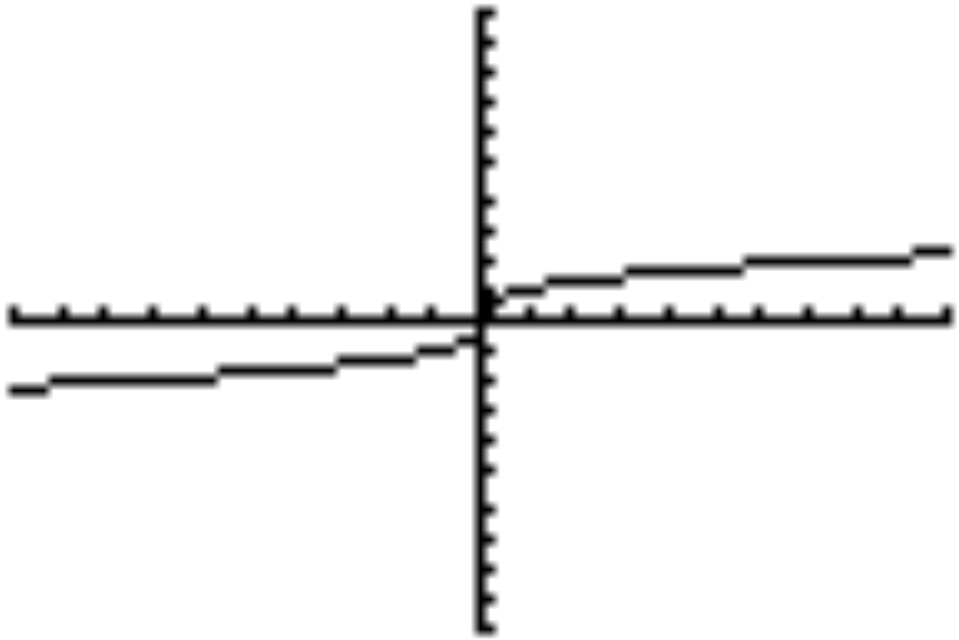
Domain:

Range:

p(0)=

p(2)=

p(100)=



A Beginning Library of Elementary Functions

The cube function; $g(x) = |x|$

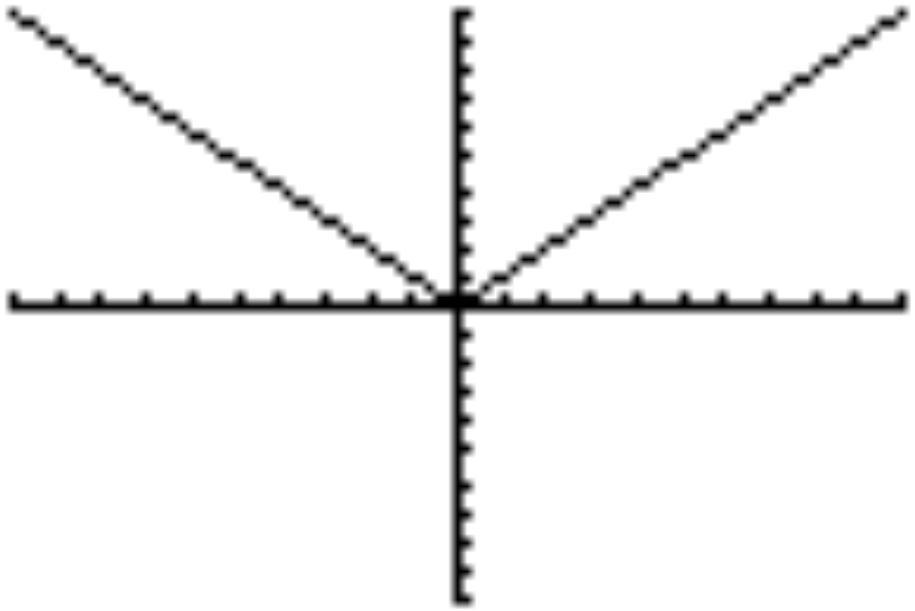
Domain:

Range:

g(0)=

g(2)=

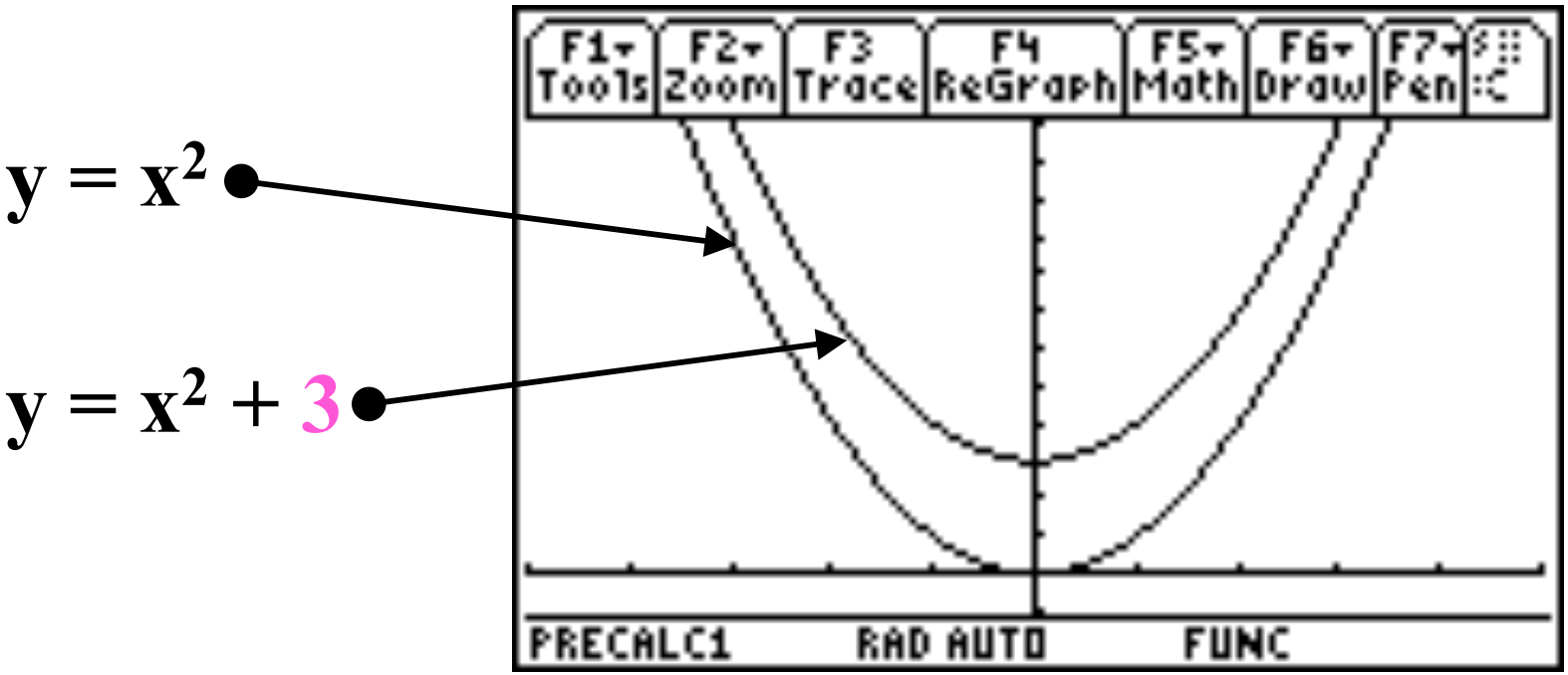
g(-2)=



Vertical Shifts

In the following discussions it is assumed that f (x) is from the previous library although the discussion is true for all functions.

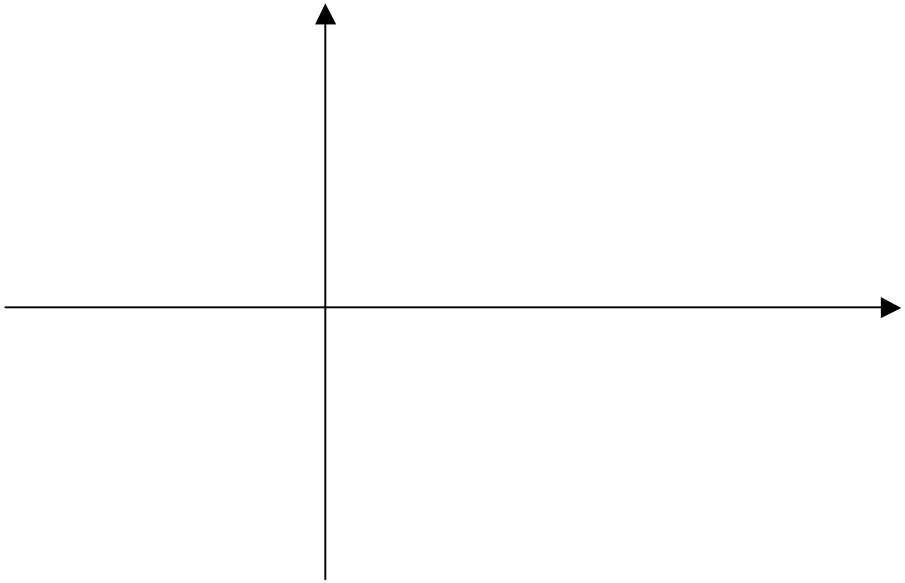
The graph of y = f (x) + k is the same as that of y = f (x) except it is shifted vertically k units. If k is positive the shift is up and if k is negative the shift is down.



Vertical Shifts

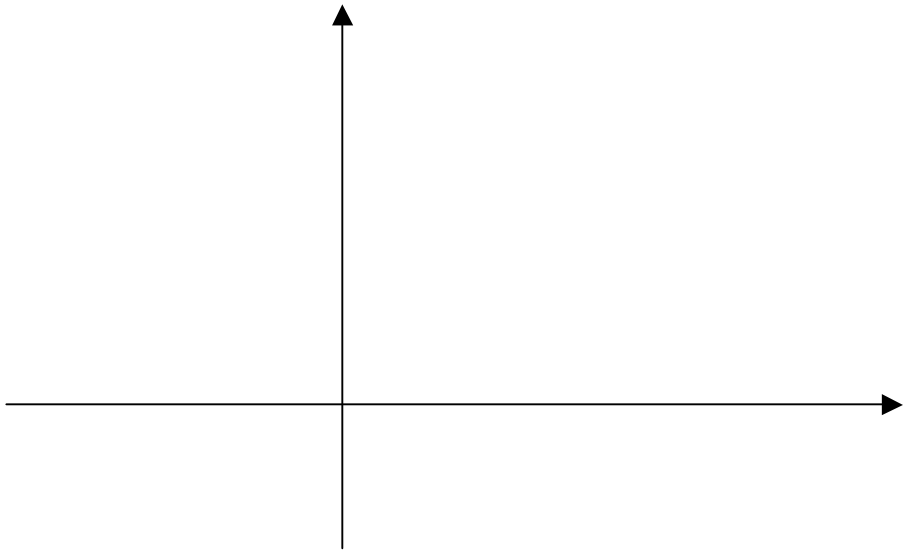
$y = x$

$y = x - 3$



$y = x^3$

$y = x^3 + 2$



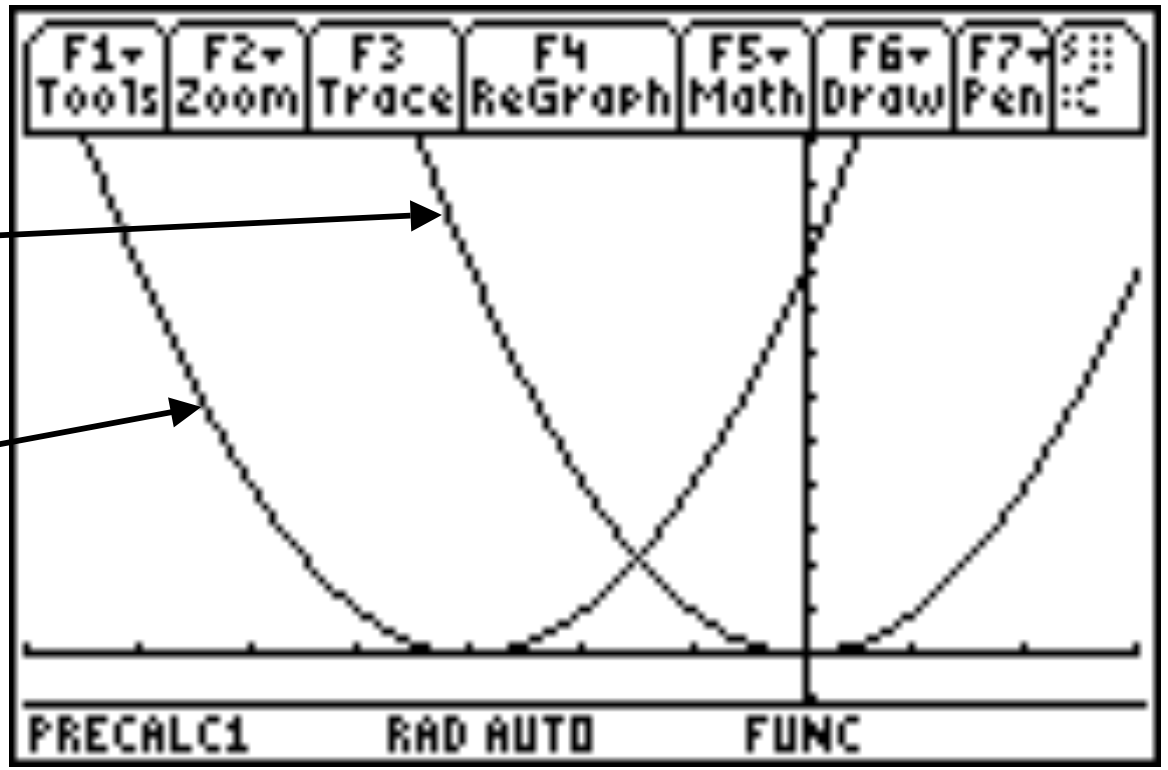
Horizontal Shifts

In the following discussions it is assumed that f (x) is from the previous library although the discussion is true for all functions.

The graph of $y = f(x + h)$ is the same as that of $y = f(x)$ except it is shifted horizontally $-h$ units. I.e. if h is positive the shift is to the left and if h is negative the shift is to the right.

$y = x^2$

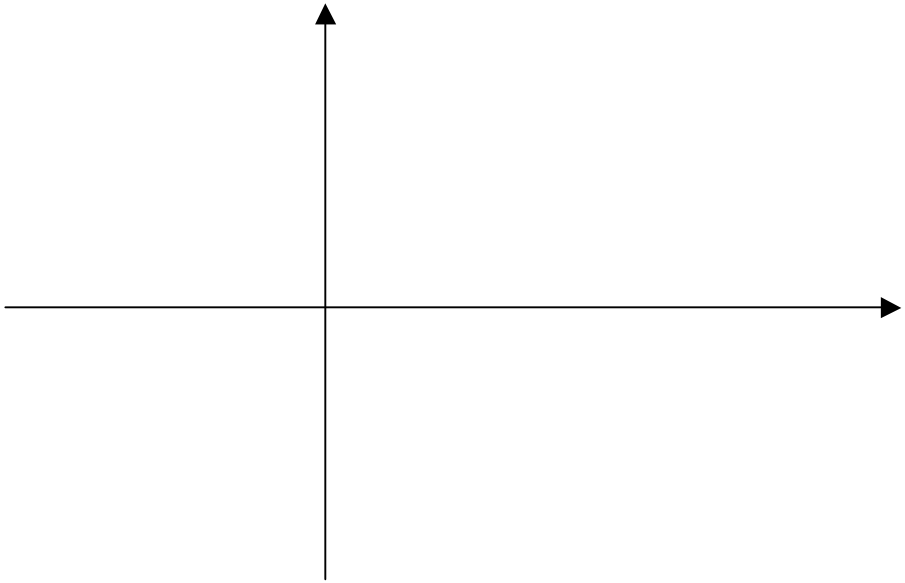
$y = (x + 3)^2$



Horizontal Shifts

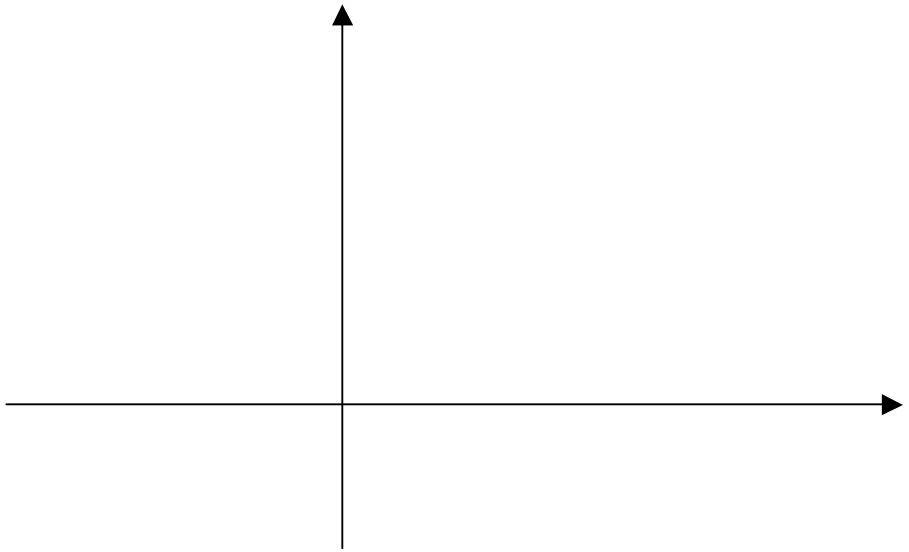
$y = x$

$y = (x - 3)$



$y = x^3$

$y = (x + 2)^3$

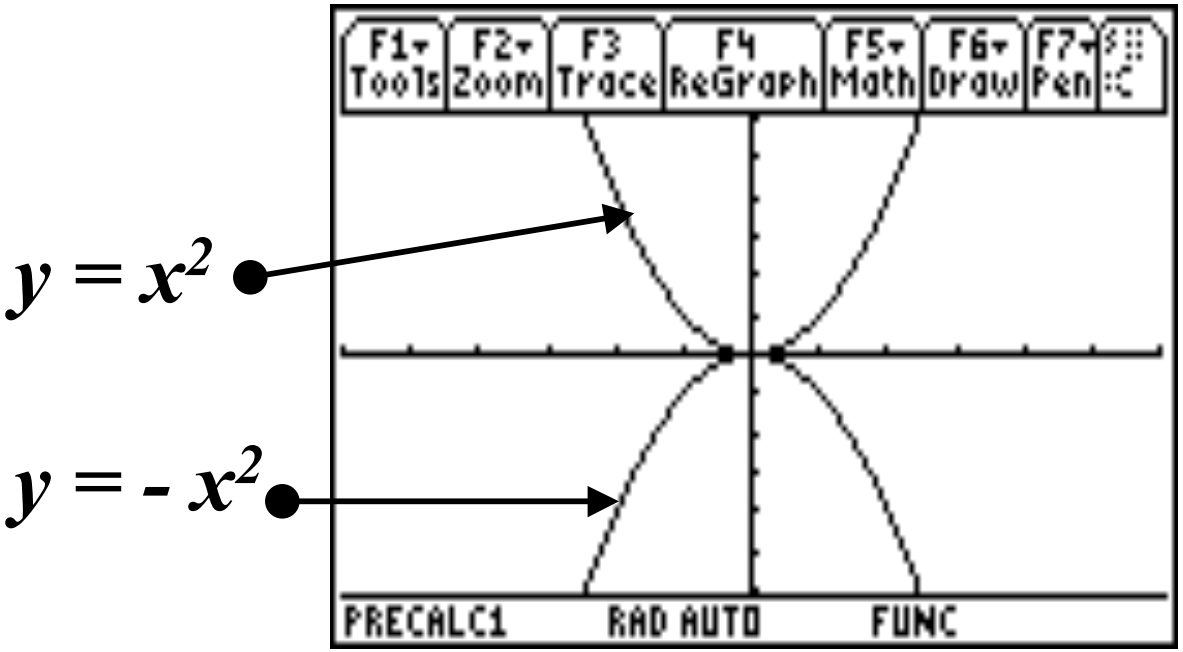


Reflections

In the following discussion, it is assumed that *f (x)* is from the previous library although the discussion is true for all functions.

Consider the graph of $y = a\, f(x)$ in comparison to $y = f(x)$: there are three cases.

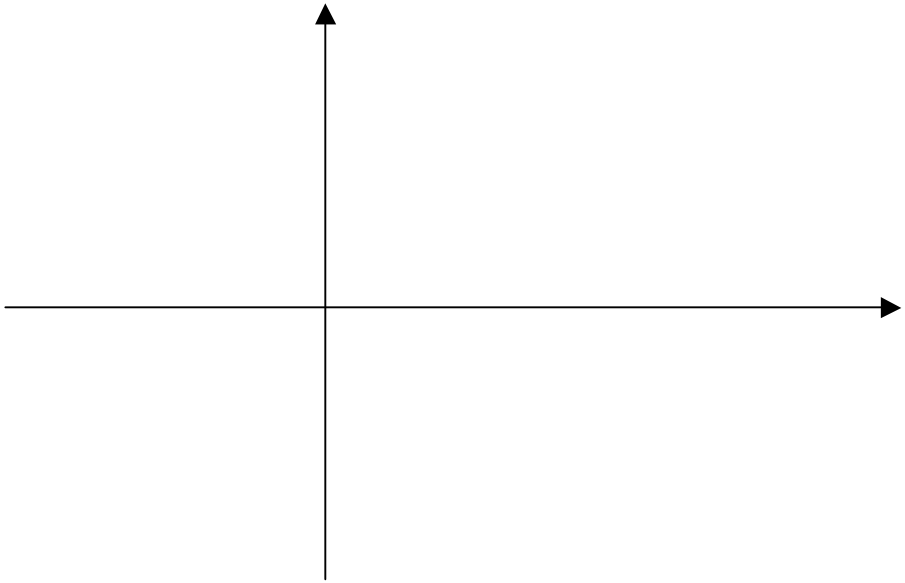
<u>Case 1</u>: If a is -1 then $y = -1\, f(x)$ is an *x* axis reflection of $y = f(x)$.



Reflections

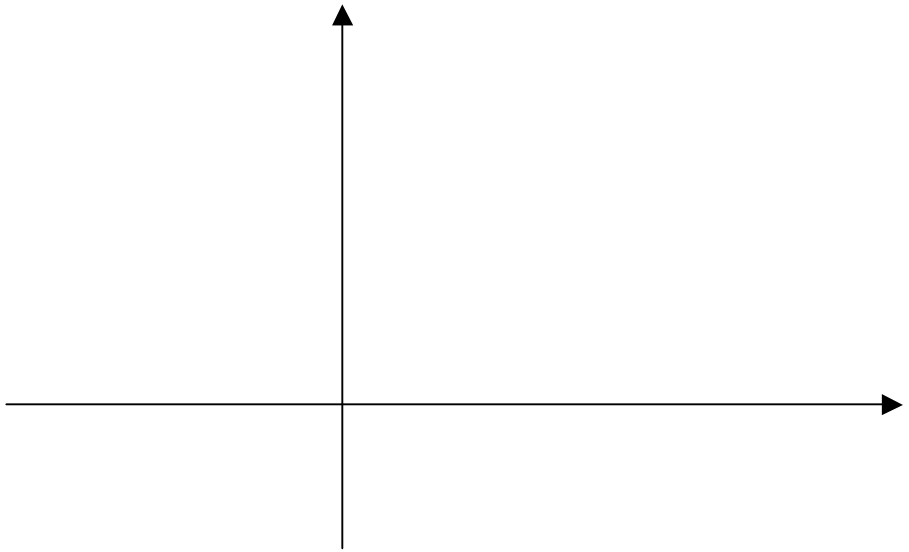
$y = x$

$y = -x$



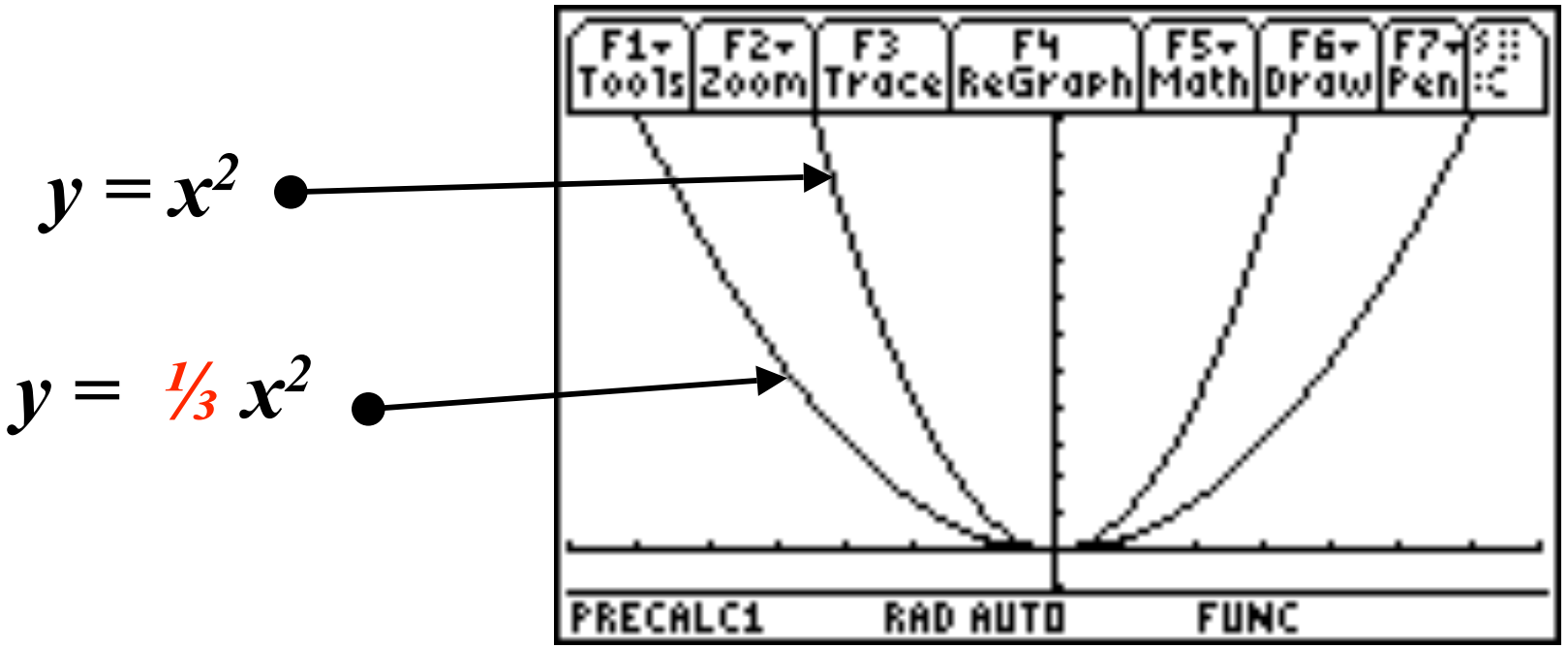
$y = x^3$

$y = -x^3$



Contractions

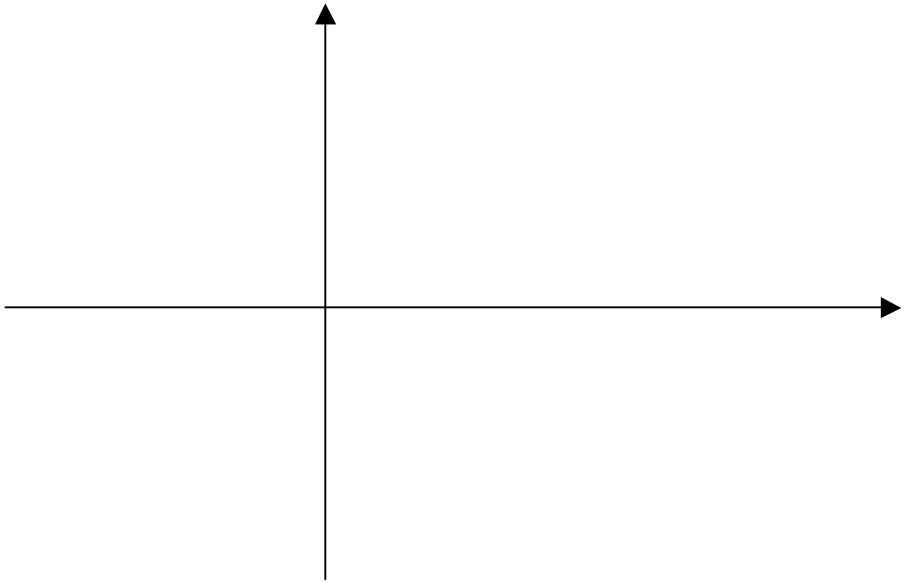
Case 2: If | *a* | is less than 1 then *y* = *a f (x)* looks like *y* = *f (x)* but has a vertical contraction (squish) by multiplying each *y* value by *a*.



Contractions

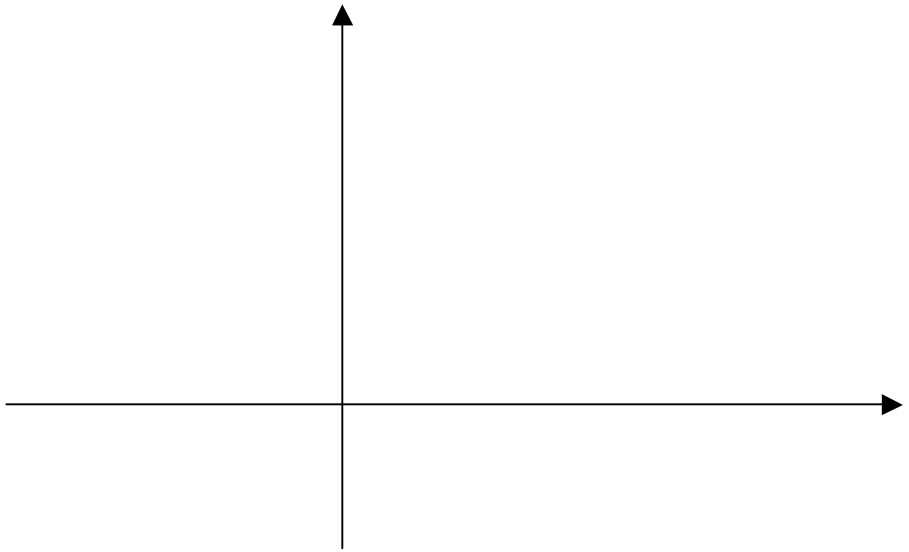
$y = x$

$y = (1/2)x$



$y = x^3$

$y = (1/2)x^3$

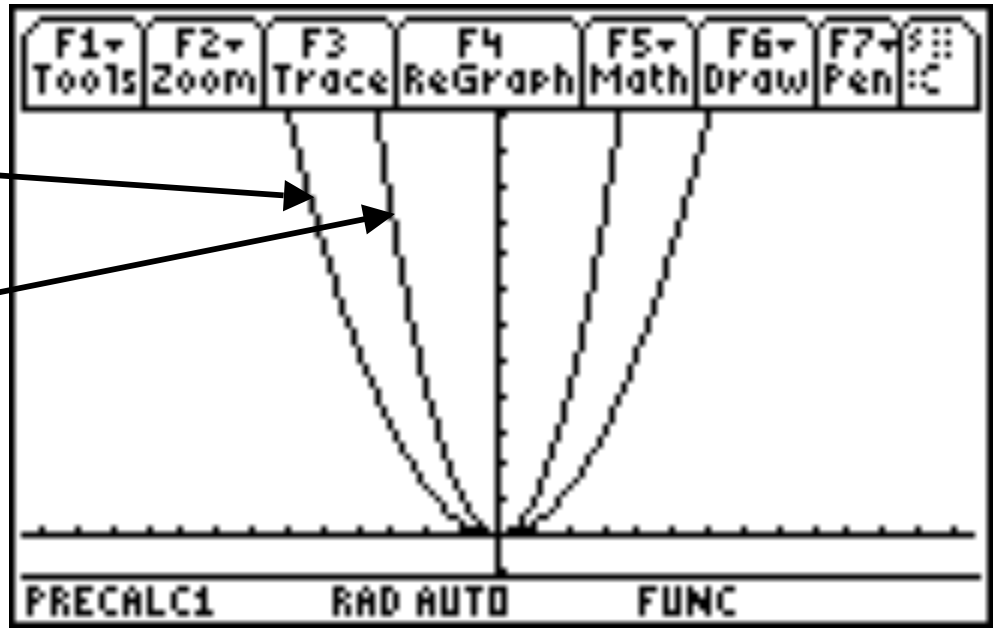


Expansions

<u>Case 3</u>: If | *a* | is greater than 1 then *y* = *a f (x)* looks like *y* = *f (x)* but has a vertical expansion (stretch) by multiplying each *y* value by *a*.

$y = x^2$

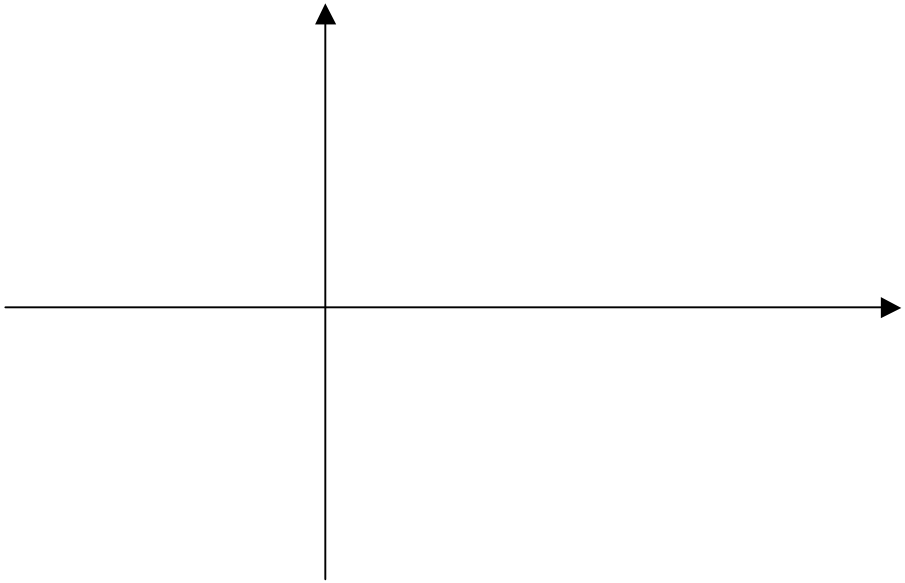
$y = 3x^2$



Expansions

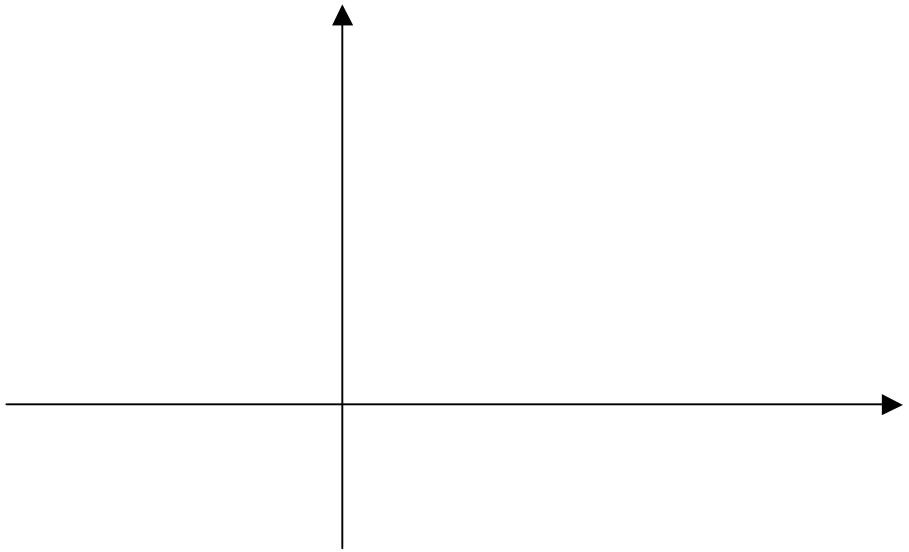
$y = x$

$y = 3x$



$y = x^3$

$y = 2x^3$

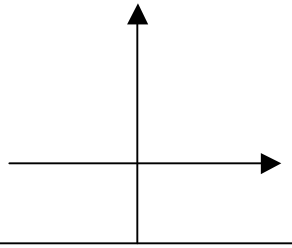


Summary

Vertical Translation:

$y = f(x) + k$

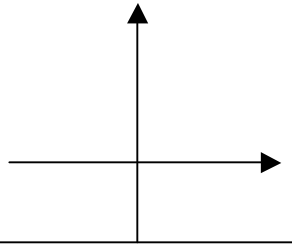
$k > 0$ shifts $f(x)$ up k units.
$k < 0$ shifts $f(x)$ down k units.



Horizontal Translation:

$y = f(x + h)$

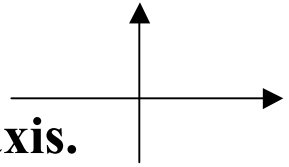
$h < 0$ shifts $f(x)$ _____ h units.
$h > 0$ shifts $f(x)$ _____ h units.



Reflection:

$y = -f(x)$

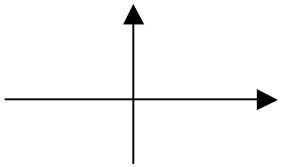
_____ the graph of $f(x)$ in the x axis.

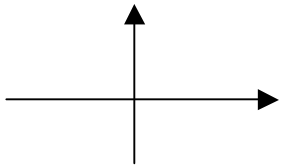


Vertical Expansion/Contraction:

$y = a\,f(x)$

$a > 1$ _____ $f(x)$ vertically by a multiple of a.
$0 < a < 1$ _____ $f(x)$ vertically by a multiple of a.





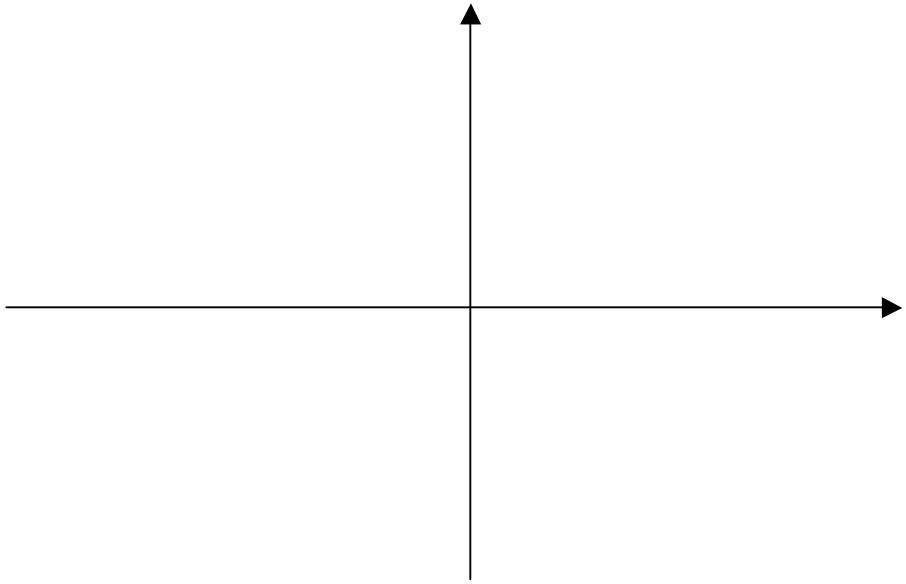
Combined Transformations

The above transformations may be combined.

$$y = a\,f(x + h) + k$$

$$y = x^2$$

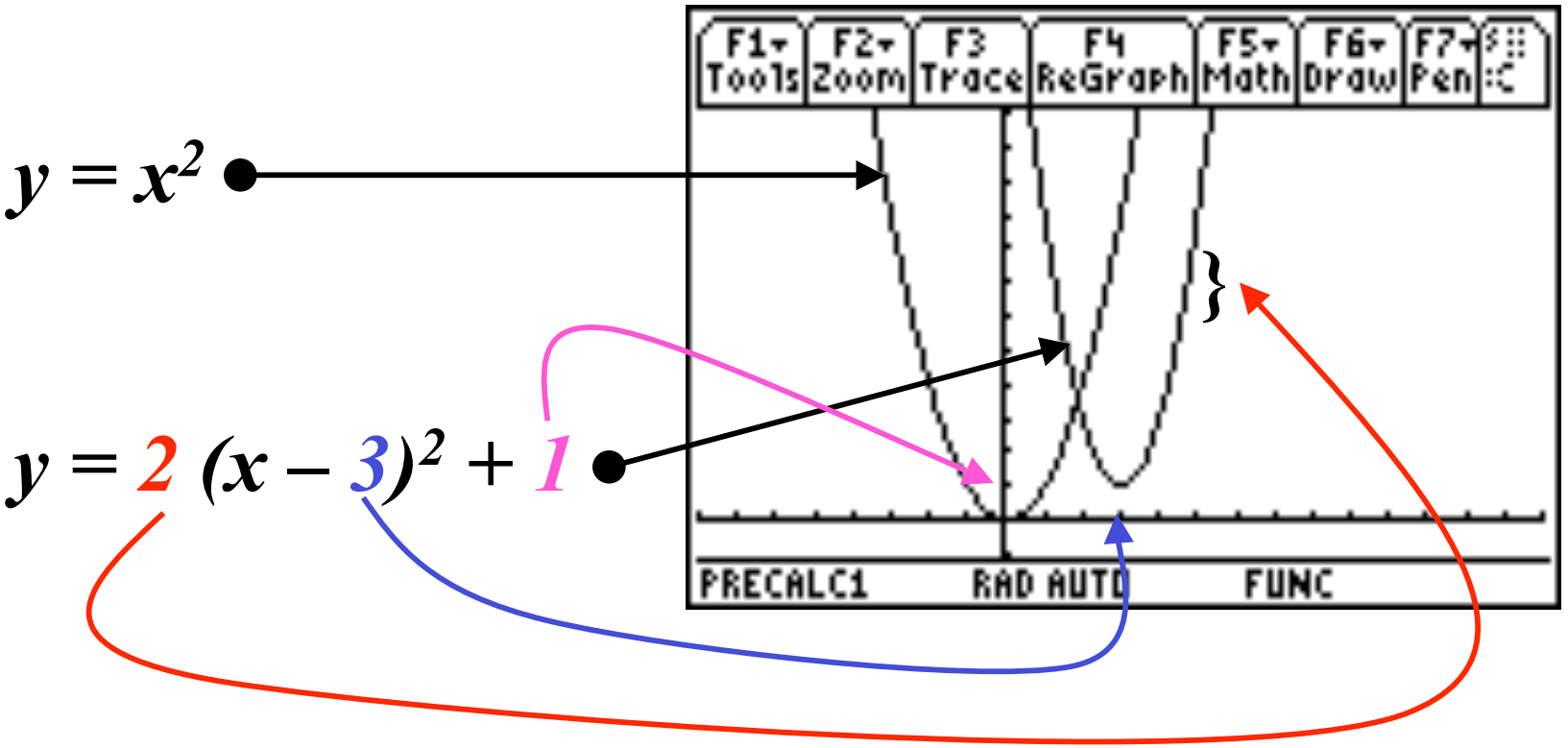
$$y = 2\,(x - 3)^2 + 1$$



Combined Transformations

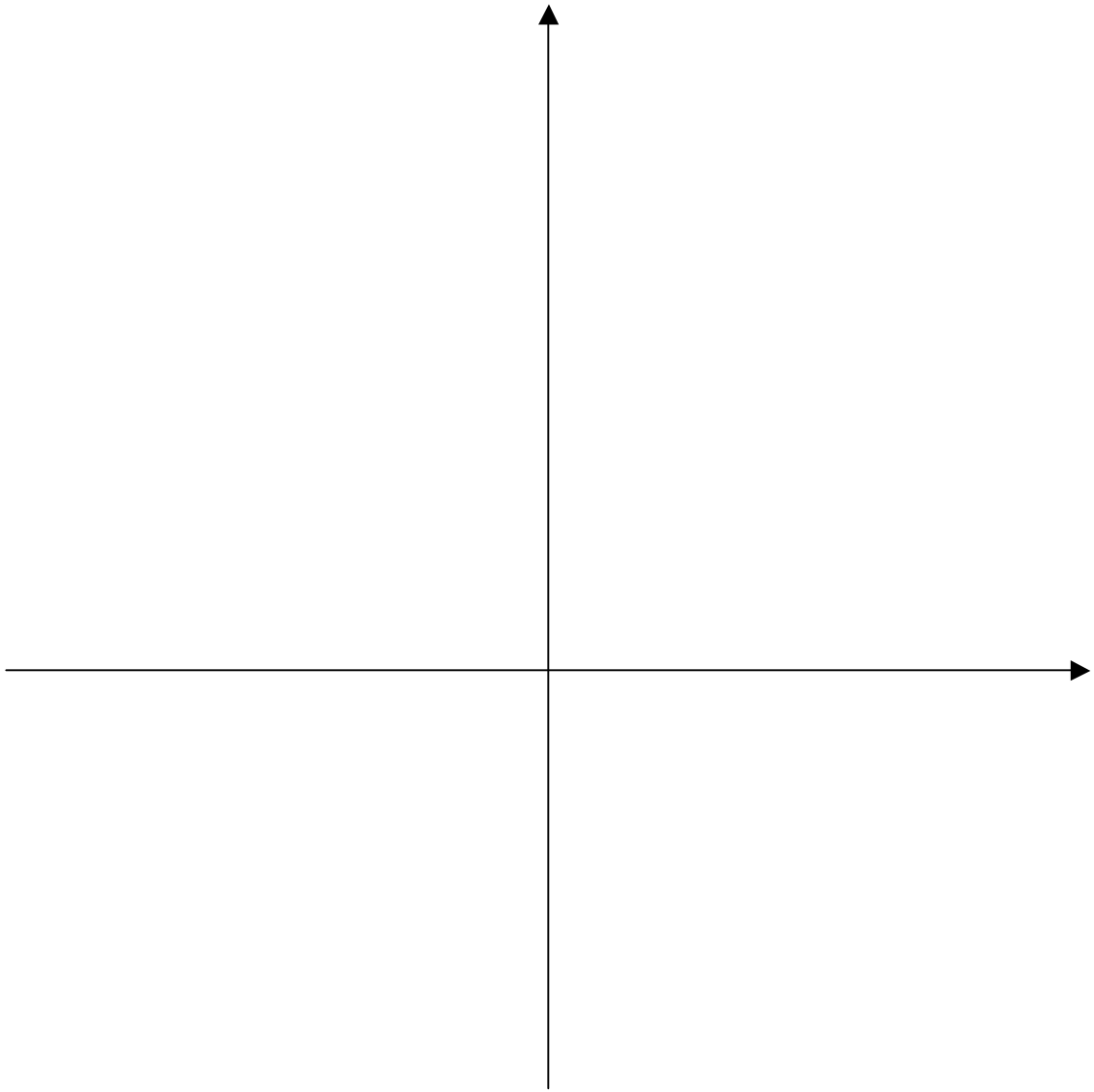
The above transformations may be combined.

$$y = a\,f(x + h) + k$$



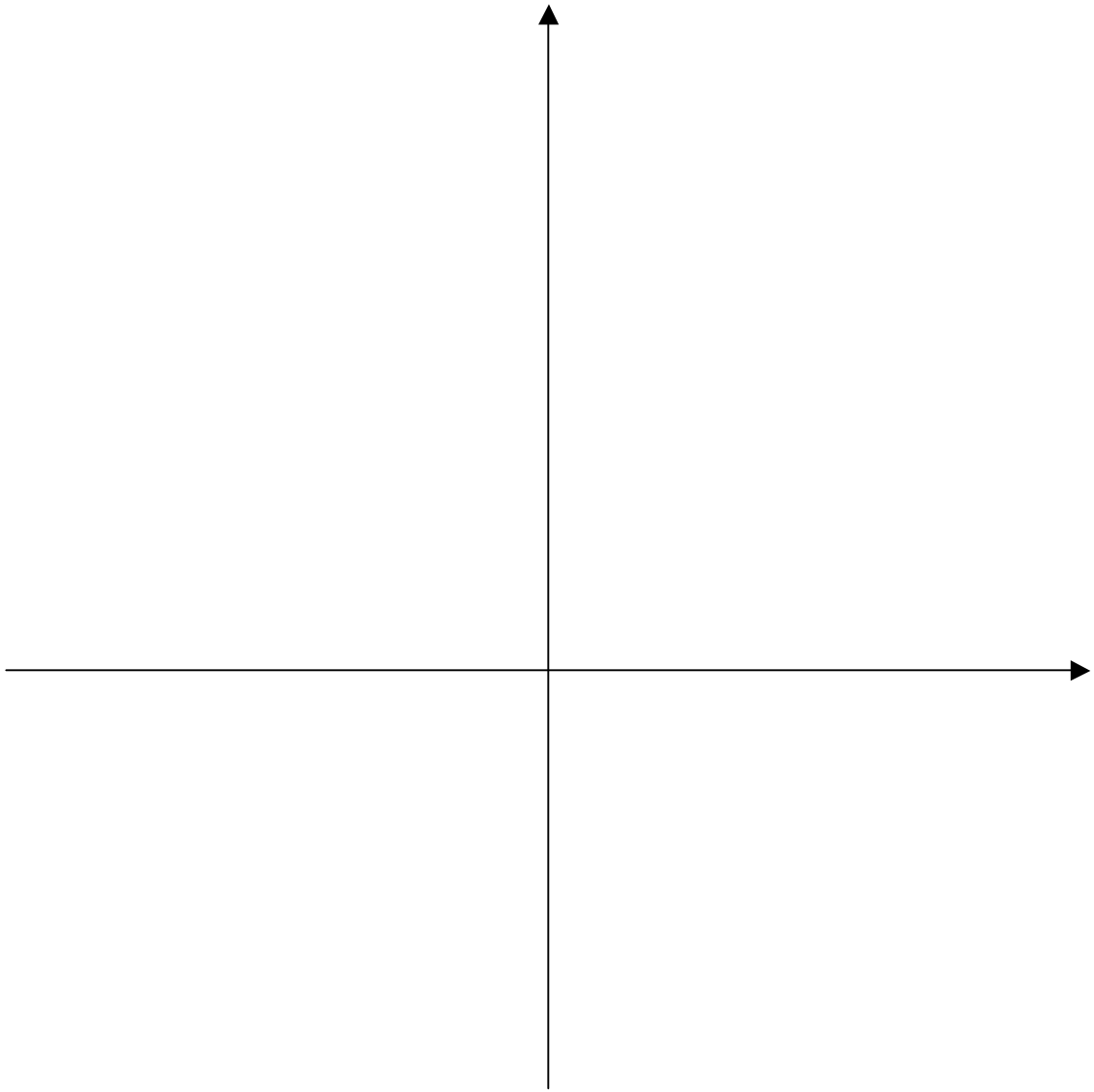
Put it all together

$y = -(x-2)^2 + 3$



Put it all together

$y = (\frac{1}{3})(x+2)^3 - 1$



Piecewise Functions

The Trussville Utiliities uses the rates shown in Table 2 to compute the monthly cost of natural gas for residential customers. Whrite a piecewise definition for the cost of consuming *x* CCF of natural gas and graph the function.

Cost per CCF	Per CCF for
$0.7	The first 50 CCF
$0.6	The next 150 CCF
$0.5	All CCF over 200CCF