Introduction to Functions
Section 2.1

- Notation
- Evaluation
- Solving
- Unit of measurement

Introductory Example: Fill the gas tank

Your gas tank holds 12 gallons, but right now you're running on empty. As you pull into the gas station, the engine sputters and dies—the gas tank is completely empty. You pump 12 gallons into the tank and swipe your credit card. How much does it cost?

That depends on the price of gas, of course! But exactly how does it depend on the price?

If the price per gallon is \$3.40, what is the cost to fill the tank?

If the price per gallon is \$4.10, what is the cost to fill the tank?

Fill the gas tank: function and symbolic form

The symbolic way to state exactly how the cost to fill the tank depends on the price per gallon is to write an algebraic expression for the cost in terms of the price. In our example:

$$C(p) = 12p.$$

Terminology (function): The cost, $C(p)$ to fill the tank is a **function** of the price p per gallon.

The symbolic form of the statement "the cost of filling the tank at a price of \$3.40 per gallon is \$40.80," is $C(3.40) = 40.80$.

Our purpose here is to practice translating statements in words into symbolic form using function notation.

Fill the gas tank

Here are some other examples from the gas tank situation:

Words: At a price of \$2.00 per gallon, it costs \$24.00 to fill the tank.
Write the symbolic form:

Words: The cost to fill a 12-gallon tank at a price of p dollars per gallon is 12 times p.
Write the symbolic form:

Words: What is price per gallon if it costs \$45.00 to fill the tank?
Symbolic form: What is p if $C(p) = 45.00$?
Solve $12p = 45.00$ for p.

Evaluate a function

Evaluate the function $C(p) = 12p$ at $p = 3.50$.
Plug $p = 3.50$ into the expression $12p$.
Given the input $p = 3.50$, determine the output $C(p)$.

$C(3.50) = 12 \times 3.50 = \$42.00$

Evaluate the function $C(p) = 12p$ at $p = 2.00$

Evaluate the function $C(p) = 12p$ at $p = 3.00$

Solve an equation

Given a cost $C(p)$, say $C(p) = \$45.00$, find the price p at which the cost is \$45.00.
Which value of p can you plug into the expression $12p$ so that $12p = 45.00$?
Given the output $C(p) = \$45.00$, what is the input p?

If we want to know what price per gallon results in a cost of \$45.00 to fill the tank, we must solve $12p = 45.00$ for p:

$$\begin{aligned} 12p &= 45.00 \\ p &= 45.00/12 \\ &= 3.75. \end{aligned}$$

Summary: If the cost to fill the tank is \$45.00, then the price per gallon is \$3.75.

Units of measurement

In the gas tank example, the price of gas p is measured in dollars per gallon and the cost to fill the tank $C(p)$ is measured in dollars. These are the **units of measurement** in this example. In other business examples the units of measurement could be money measured in thousands of dollars, number of television sets produced, square feet in a house, or kilowatt hours. It all depends on the business setting. You should always state your results using the proper units of measurement.

Goals:

1. use function notation to make statements about business situations

2. use functional notation to solve problems in business settings

3. summarize the results of our symbolic (algebraic) manipulations into statements in a business setting

4. use proper units of measurement for the functions and statements.

Electricity costs

Edcon power company charges its residential customers \$14.00 per month plus \$0.10 per kilowatt-hour (KWH) of electricity used. Thus, the monthly cost for electricity is a function of the number of KWHs used. In symbols, let k be the number of KWHs used in a month, and $E(k)$ be the monthly cost for electricity in dollars.

- What are the units of measurement for k and for $E(k)$?

- Write the symbolic form for the statement: The monthly cost for using 800 kilowatt-hours of electricity is \$94.00.

Electricity costs

Edcon power company charges its residential customers \$14.00 per month plus \$0.10 per kilowatt-hour (KWH) of electricity used. Thus, the monthly cost for electricity is a function of the number of KWHs used. In symbols, let k be the number of KWHs used in a month, and $E(k)$ be the monthly cost for electricity in dollars.

- Write the symbolic statement $E(660) = 80$ in words.
- Write the symbolic form for the statement: The monthly cost for the 101st KWH is \$0.10.
- Write the symbolic form for the statement: The monthly cost for using k KWHs is \$100.00.

Electricity costs

We can give a formula for the electricity costs as a function of kilowatt-hours used as follows:

$$E(k) = 14.00 + 0.10k. \tag{1}$$

The 14.00 in the formula is the amount the amount of money the customer pays regardless of how many kilowatt-hours are used. Even if the customer uses no electricity, the monthly charges will still be \$14.00. This is called the **fixed cost**. In addition to the fixed cost, the customer pays \$0.10 per kilowatt-hour used. So k KWHs would cost an additional $\$0.10k$. Adding these two types of costs, we get a total cost of $14.00 + 0.10k$ dollars per month.

The formula above for $E(k)$ allows us to calculate the monthly cost for any number of KWHs used. If the customer uses 200 KWHs, then $k = 200$ and the cost is

$$E(200) = 14.00 + 0.10(200) = 14.00 + 20.00 = 34.00.$$

Summary: The monthly cost for using 200 KWHs is \$34.00.

One of the reasons for writing statements in symbolic form is that it permits us to manipulate algebraic expressions in order to solve problems. For example,

How many KWHs can be used if the monthly cost is $55.00?

In this case, we do not know k, the number of KWHs used. But we do know the cost of using k KWHs, $E(k) = 55.00$. So by replacing $E(k)$ with the formula (1) we can write $E(k) = 55.00$ as

$$14.00 + 0.10k = 55.00.$$

Now the problem is to solve the above equation using algebra. The answer is $k = 410$.

Summary: If the monthly cost is $55.00, then the number of KWHs used is 410.

Problems on electricity costs

$$E(k) = 14.00 + 0.10k$$

• Compute the monthly cost for using 500 KWHs and write a summary statement in words using the proper units.

• Compute the monthly cost for using 600 KWHs and write a summary statement in words using the proper units.

•How many KWHs can be used if the monthly cost is $50.00? Translate this problem into symbolic form as an algebra problem.

Dippin Donuts

Dippin Donuts makes and sells donuts to some local restaurants and to individual customers who come to the shop. Depending on the day of the week and the size of the orders from local restaurants, the owner of Dippin Donuts decides how many donuts to make for the day.

How much does it cost to make the donuts? The baker shows up for work at 4:00 in the morning to make the donut batter and start the donut cooker. He gets \$45.00. In addition, the batter and oil costs \$0.12 per donut. Thus the total cost to make the donuts for the day is a function of the number of donuts that are made. So letting x stand for the number of donuts made, and $C(x)$ for the total cost to make x donuts, the formula for $C(x)$ is

$$C(x) = 45.00 + 0.12x,$$

where the **unit of measurement** for x is donuts and for $C(x)$ is dollars.

Dippin Donuts

$$C(x) = 45.00 + 0.12x,$$

Problem: How much does it cost to make 220 donuts?

The problem is to evaluate $C(x)$ when $x = 200$:

$$C(220) = 45.00 + 0.12(220) = 71.40.$$

Summary: The cost to make 220 donuts is \$71.40.

Dippin Donuts

$$C(x) = 45.00 + 0.12x,$$

Problem: How many donuts can be made for a cost of $120.00?

This time x is unknown. We want to find the value of x so that the cost is $120.00. So we need to solve the equation $C(x) = 120.00$:

$$\begin{aligned} C(x) &= 120.00 \\ 45.00 + 0.12x &= 120.00 \\ 0.12x &= 120.00 - 45.00 \\ &= 75.00 \\ x &= \frac{75.00}{0.12} \\ &= 625. \end{aligned}$$

Summary: 625 donuts can be made for a cost of $120.00.

Dippin Donuts

$$C(x) = 45.00 + 0.12x,$$

- Problem: How much does it cost to make 400 donuts?

Summary:

- Problem: How many donuts can be made for a cost of $180.00?

Summary:

Functions: Part 2
Section 2.1

- Definition
- Graphs
- General cost and demand functions

Graphing: point-by-point

Sketch a graph of $y = 2x - 1$.

Construct a table of values that satisfy the equation for the function.

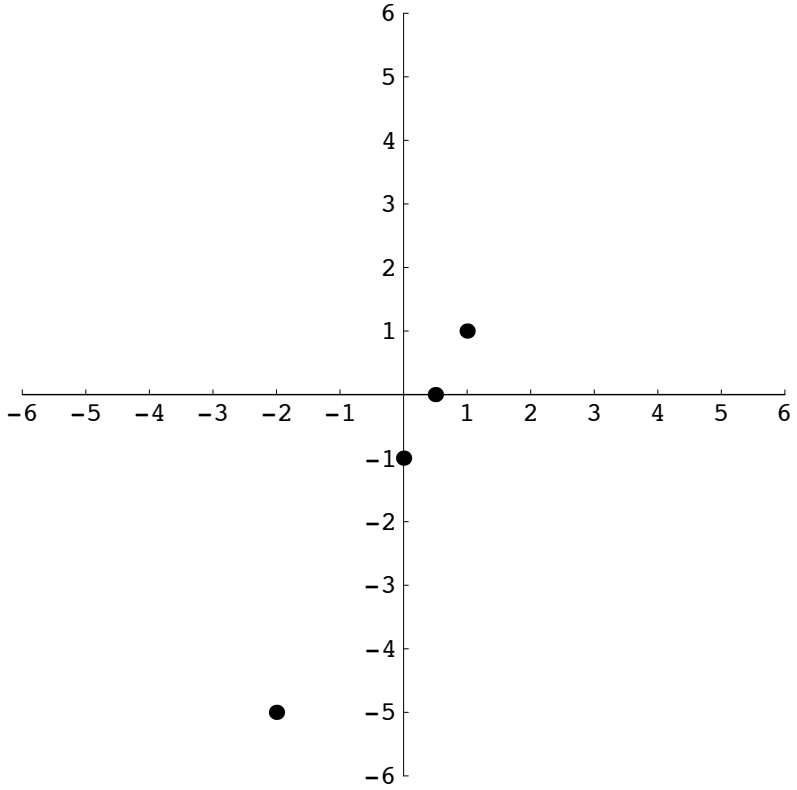
x	y
-2	
0	
$1/2$	
1	

Graphing: point-by-point

Sketch a graph of $y = 2x - 1$.

Construct a table of values that satisfy the equation for the function.

x	y
-2	-5
0	-1
$1/2$	0
1	1



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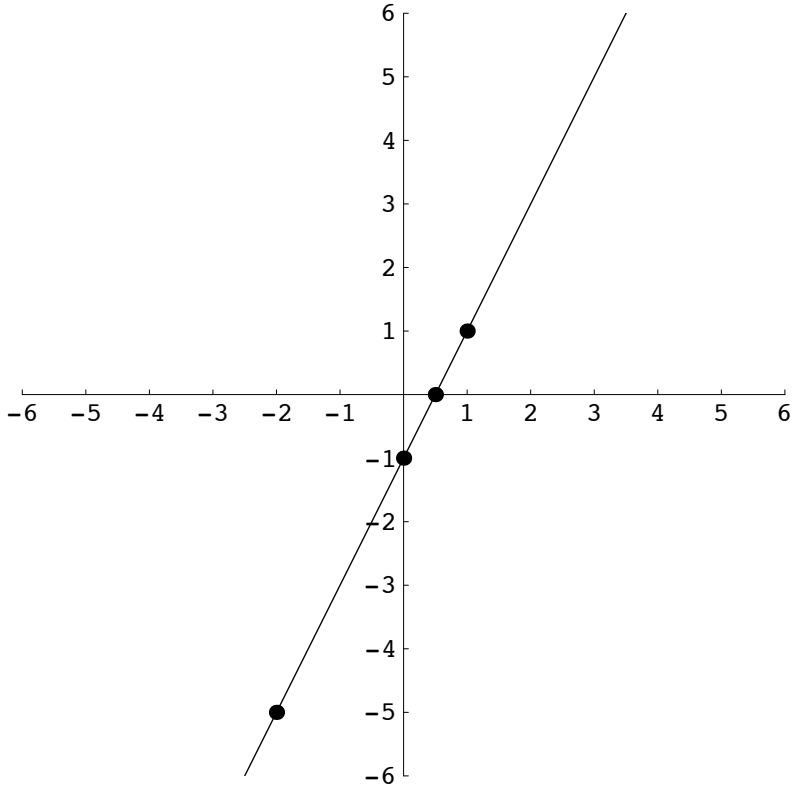


Function Notation

$$y = 2x - 1 \qquad f(x) = 2x - 1$$

$$\begin{aligned} f(x) &= 2x - 1 \\ f(2) &= \\ f(-1) &= \\ f(0) &= \\ f(2/3) &= \end{aligned}$$

Function Notation

$$y = 2x - 1 \qquad f(x) = 2x - 1$$

$$\begin{aligned} f(x) &= 2x - 1 \\ f(2) &= 2 \times 2 - 1 = 3 \\ f(-1) &= \\ f(0) &= \\ f(2/3) &= \end{aligned}$$

Definition of a Function

Examples:

For each sale of x items there corresponds a revenue.

To each legal automobile driver, there is a driver's license number.

To each student in Math 103, there corresponds a grade in the course.

To each number x, there corresponds $2x$, its double.

Definition: A function is a rule that produces a correspondence between two sets of objects (usually numbers) such that each object in the first set (called the domain of the function) corresponds to exactly one object in the second set (called the range of the function).

Functions Determined by Equations
Graphs of Functions

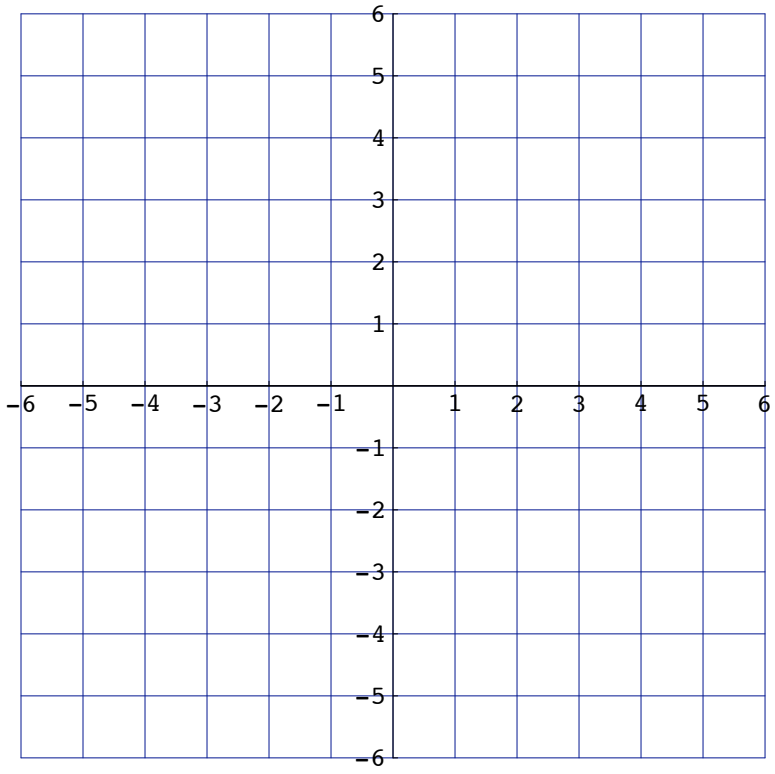
Both the domain and the range are sets of numbers.

x is usually used for the number in the domain

y is usually used for the number in the range.

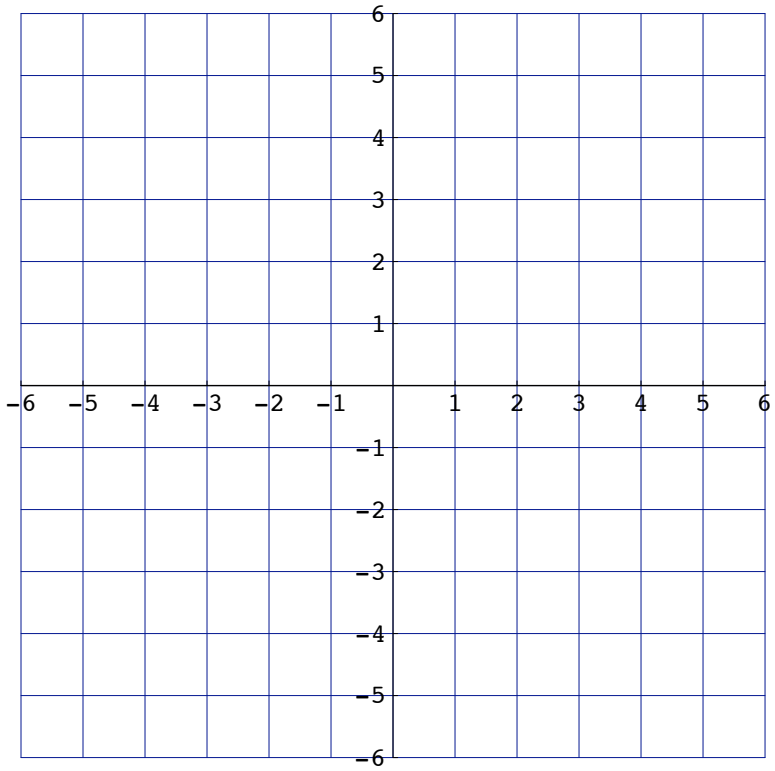
Examples:

$y = 2x - 1,\ f(x) = 2x - 1$



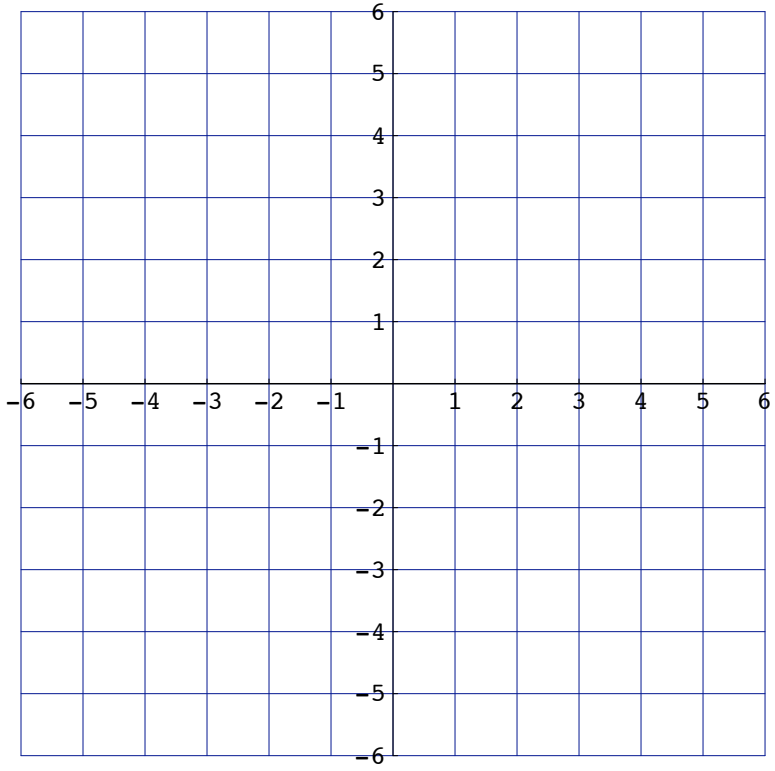
Functions Determined by Equations
Graphs of Functions

Example: $y = \dfrac{1}{x},\ f(x) = \dfrac{1}{x}$



Functions Determined by Equations
Graphs of Functions

Example: $y = x^2 - 1, \ f(x) = x^2 - 1$



Function Notation

x, y equation	function notation
$y = -5x + 3$	$f(x) = -5x + 3$
$y = 2x^2 + 4x - .5$	$g(x) =$
$y = \frac{2}{x+1}$	$h(x) =$

Evaluation of a function:

$$f(3) =$$

$$f(-2) =$$

$$g(1) =$$

$$h(4) =$$

$$h(1/2) =$$

$$g(a) =$$

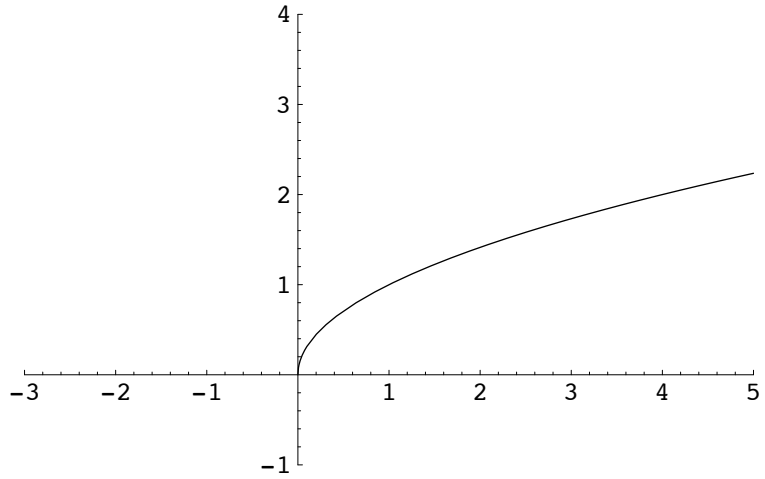
$$g(a+1) =$$

Finding the Domain of a Function

Let $f(x)$ be a function given by an equation. Sometimes $f(x)$ doesn't make sense for certain values of x.

Example: $f(x) = \sqrt{x}$.

What's $f(-3)$? Not defined.



Domain of $f(x)$:

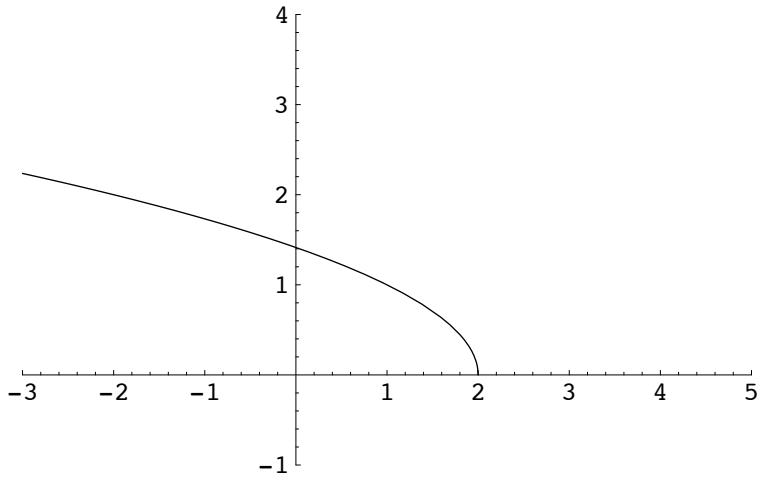
Inequality notation: $x \geq 0$

Interval notation: $[0, +\infty)$

Finding the Domain of a Function

Example:

$f(x) = \sqrt{2 - x}$



Domain of $f(x)$:

Inequality notation:

Interval notation

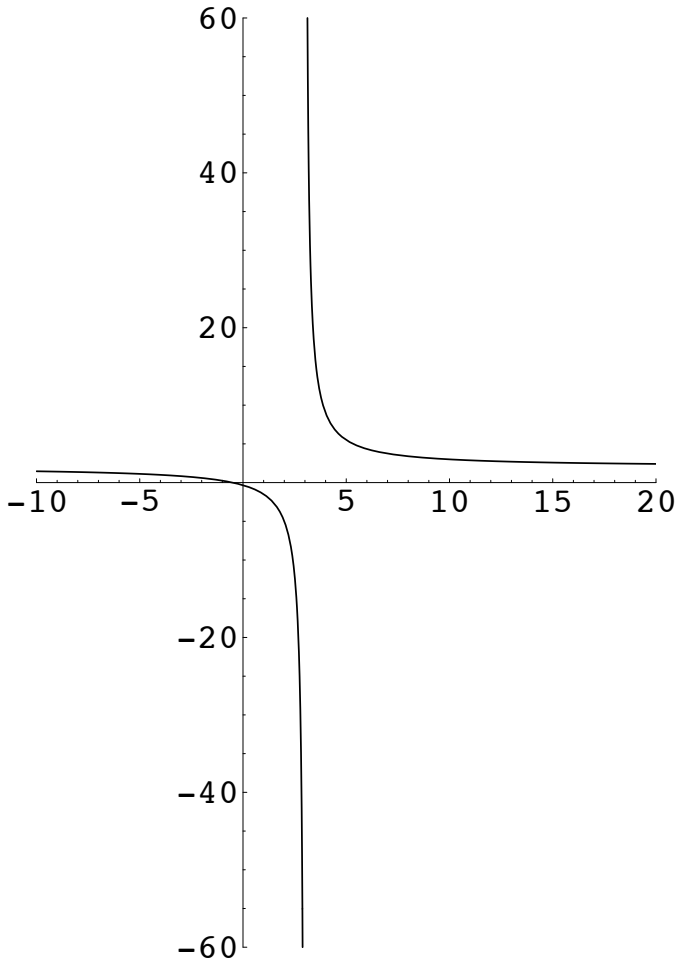
Finding the Domain of a Function

Example: $f(x) = \dfrac{2x+1}{x-3}$

Domain of $f(x)$:

The expression makes sense for all values of x EXCEPT $x = 3$.

Inequality notation: $x \neq 3$


60
40
20
-10
-5
5
10
15
20
-20
-40
-60

Cost, Price-Demand, Revenue, and Profit Functions

x is the number of units sold (independent variable)

Cost Function: $C(x)$ is the cost to produce x units.

There is a fixed cost, a and a cost per unit, b.

$C(x) = a + bx$

Price-demand Function: $p(x)$ is the price per unit. The price per unit decreases as the number of units produced increases.

$p(x) = m - nx$

Revenue Function: $R(x) = xp(x) = x(m - nx) = xm - nx^2$

Total revenue is number sold times the price per unit.

Profit Function:

$P(x) = R(x) - C(x)$

Profit is revenue minus cost.

Cost, Price-Demand, Revenue, and Profit Functions

Example: A publishing company is preparing to market a new book on turnip gardening. The author gets a flat fee of $5000 for writing the book. It costs $8300 to to typeset the book plus $18 per book to print and bind the book.

Fixed costs are $5000 + $ 8300 = $13,200.
Cost per unit is $18.

$C(x) = 13200 + 18x$

The publisher believes that the price-demand function is
$p(x) = 140.00 - .10x$

That is, starting at a sales price of $140 per copy, the price decreases by $.10 for each additional copy sold.

So

$$R(x) = x(140 - .10x) = 140x - .10x^2$$

and

$$P(x) = R(x) - C(x) = -13200 + 122x - .10x^2$$

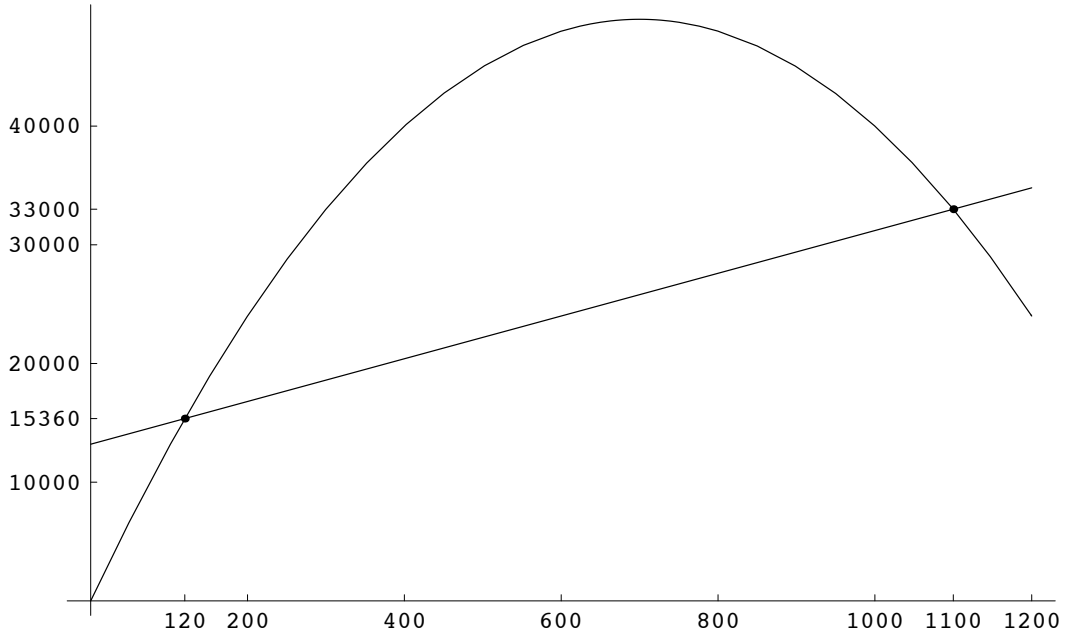
Cost, Price-Demand, Revenue, and Profit Functions

$C(x) = 13200 + 18x$

$p(x) = 140.00 - .10x$

$R(x) = x(140 - .10x) = 140x - .10x^2$

$P(x) = R(x) - C(x) = -13200 + 122x - .10x^2$



Break-even: How many units must be produced to break-even?

Solve $C(x) = R(x)$