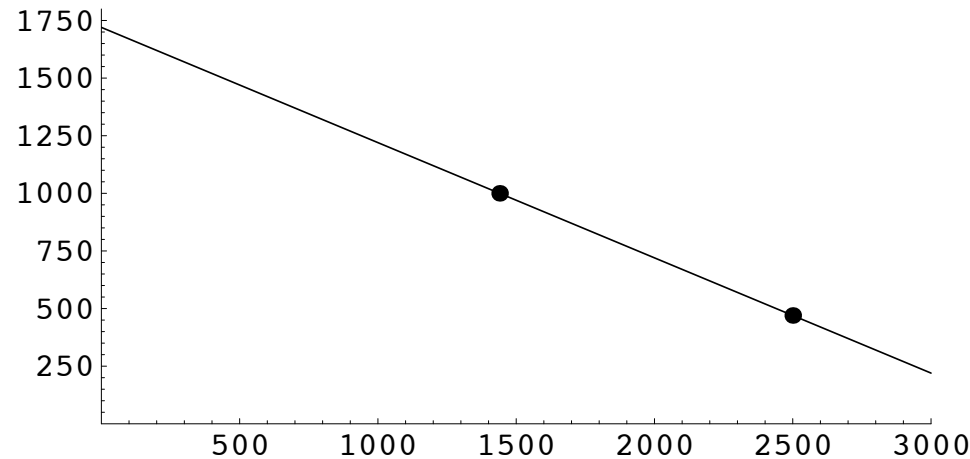


# Math 103 Section 1.2: Linear Equations and Graphing

- Linear Equations in two variables
- Graphing  $Ax + By = C$
- Slope of a line
- Special Forms of a linear equation
- More applications

The Price-demand equation again:  $d = 1720 - .50p$ . We can represent many solutions on a graph.



**Definition 1** A linear equation in two variables is an equation that can be written in the **standard form**

$$Ax + By = C$$

where  $A$ ,  $B$ , and  $C$  are constants ( $A$  and  $B$  not both zero) and  $x$  and  $y$  are variables.

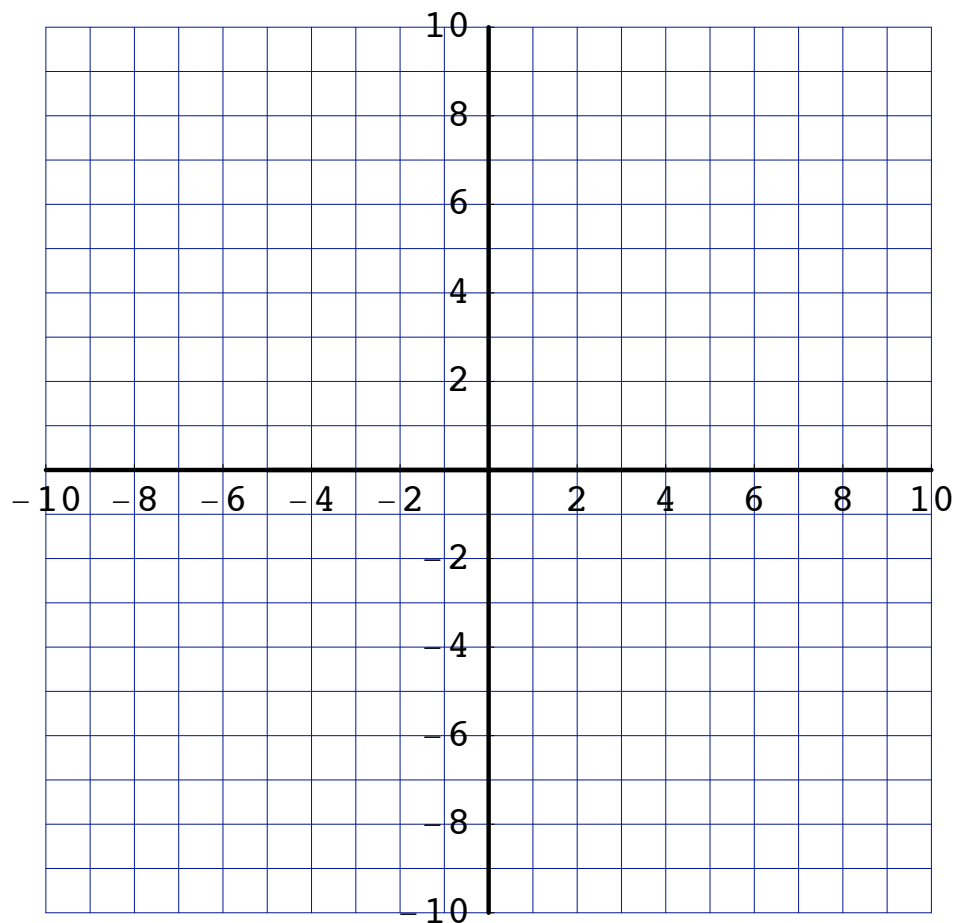
Circle the LINEAR equations:

$$x = y \quad x^2 - y = 3 \quad 3x - 4y = 6$$

$$y = \sqrt{3x - 1} \quad y = \frac{3}{4}x - \frac{3}{2}$$

$$x = 3 \quad y = -2.$$

Do you think that all such linear equations, when graphed will give a straight line? Why? Let's try some examples:



What about  $x - y = 0$ ?

What about  $x = 3$ ?

What about  $y = -1$ ?

## The shape of the graph of a linear equation:

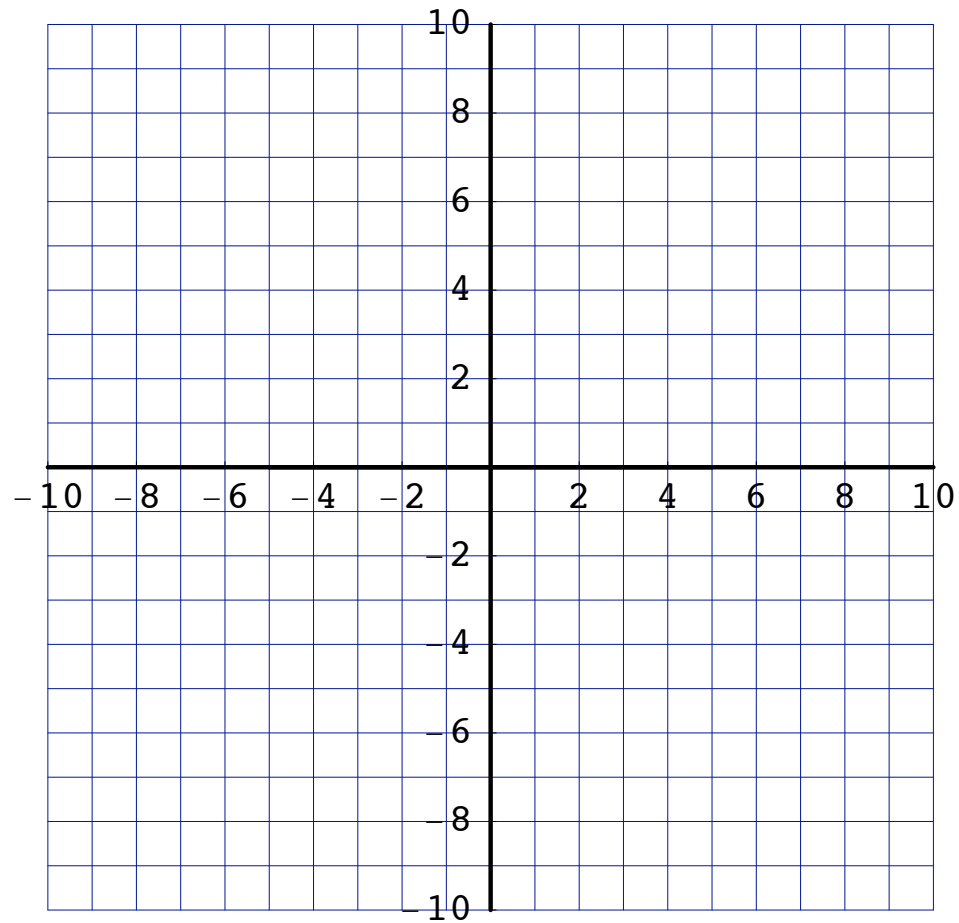
**Theorem 1** *The graph of any equation of the form  $Ax + By = C$  is a line, and any line in the cartesian coordinate system is the graph of an equation of this form.*

This theorem makes it very easy to graph a linear equation...  
For example if we know two points on the graph we are done!!  
Moral: Theorems are your friends.

Two special cases:

- Horizontal lines  $\Leftrightarrow y = c$  for some number  $c$
- Vertical lines  $\Leftrightarrow x = c$  for some number  $c$

Try graphing  $4x - 3y = 12$  by finding two easy solutions to the equation and plotting those.



What about  $x = 3$ ?

What about  $y = -4$ ?

## $x$ - and $y$ - intercepts of an equation:

**Definition 2** *In the graph of any equation of two variables, the points where the graph of an equation crosses the  $x$ -axis are called the  $x$ -intercepts and the points where the graph crosses the  $y$ -axis are called the  $y$ -intercepts.*

What do the coordinates of an  $x$ -intercept look like?

To find them set  $y = 0$  and solve for  $x$ .

What do the coordinates of a  $y$ -intercept look like?

To find them set  $x = 0$  and solve for  $y$ .

The intercepts of a linear equation are easier to locate than the intercepts of most other equations.

## $x$ - and $y$ -intercepts of a linear equation:

Find the  $x$ - and  $y$ - intercepts for  $4x - 3y = 12$ .

Find the  $x$ - and  $y$ -intercepts for  $7x - .2y = 12$ .

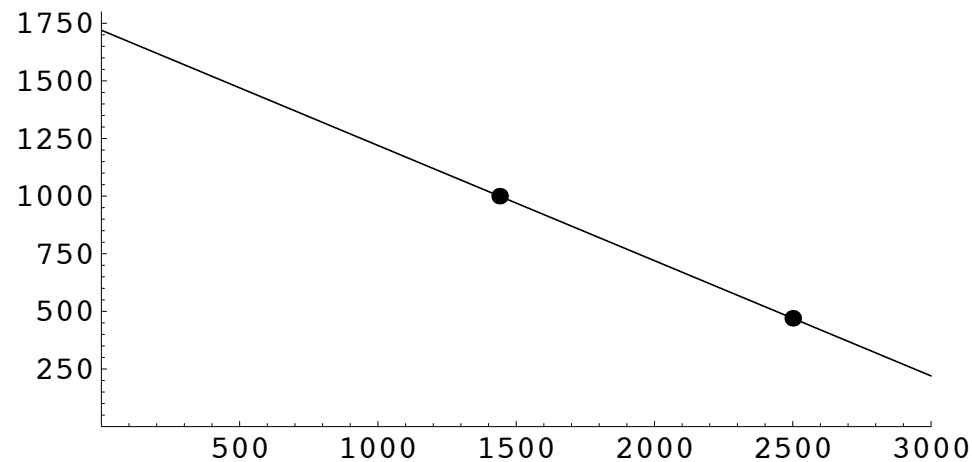
Could a linear equation have more than one  $x$ -intercept?

Does a linear equation always have an  $x$ -intercept? CAREFULL  
HERE!

## Intercepts for the Price-demand equation.

$$d = 1720 - .50p$$

What are the  $p$ - and  $d$ -intercepts? What do they mean here?



## The slope of a linear equation: the price-demand equation

$$d = 1720 - .50p$$

If price increases by \$1 how much does demand decrease?

Does this depend on the starting price?

If price increases by \$1000 how much does demand decrease?

Does this depend on the starting price?

Is the decrease in demand *always* .50 times the increase in price?

## Slope of a line:

The slope or steepness of a straight line is the same between any two points on the line. We can reformulate this property of a line in algebraic terms as follows:

For any two points  $(x_1, y_1)$ ,  $(x_2, y_2)$  on the line, the ratio of the change in  $y$  (change in  $y$  is  $y_2 - y_1$ ) to the change in  $x$  ( $x_2 - x_1$ ) is the same no matter which two points on the line are chosen. This common ratio is the slope of the line.

**Definition 3** *The slope of a line is defined as*

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

*where  $(x_1, y_1)$  and  $(x_2, y_2)$  are ANY two points on the line.*

What is the slope of the line defined by the linear equation  $4x - 3y = 12$ ?

What is the slope of the line defined by the linear equation  $d = 1720 - .50p$ ?

## Slope of the special cases: vertical and horizontal lines

Slope formula:  $m = \frac{y_2 - y_1}{x_2 - x_1}$  where  $(x_1, y_1)$  and  $(x_2, y_2)$  are ANY two points on the line.

What is the slope of the line defined by the linear equation  
 $y = -4$ ?

What is the slope of the line defined by the linear equation  
 $x = 3$ ?

## Special forms for linear equations: Slope-Intercept form.

**Definition 4** *An equation of the form*

$$y = mx + b$$

*is a linear equation in **slope-intercept form**.*

Put  $4x - 3y = 12$  into slope-intercept form.

What are  $m$  and  $b$ ?

What do they mean?

Price-demand equation:  $d = 1720 - .50p$ .

Put this into slope-intercept form.

What are  $m$  and  $b$ ?

What is the meaning of  $m$ ? of  $b$ ?

In general, the graph of an equation  $y = mx + b$  defines a line with slope  $m$  and  $y$ -intercept  $(0, b)$ .

## Special forms for linear equations: Point Slope form.

**Definition 5** *An equation of the form*

$$y - y_1 = m(x - x_1)$$

*is a linear equation in **point-slope form**.*

In general, the graph of an equation  $y - y_1 = m(x - x_1)$  is a line with slope  $m$  passing through the point  $(x_1, y_1)$ .

Example: What is the point-slope equation for the line with a slope of 3 passing through the point  $(-4, 6)$ ?

The graph of an equation  $y - y_1 = m(x - x_1)$  is a line with slope  $m$  passing through the point  $(x_1, y_1)$ .

Example: Find an equation for the line passing through the points  $(2, 1)$  and  $(5, 3)$ .

To use the point-slope form for the line, we need to know one point and the slope. In this problem the slope is not given, but two points are known. To find the slope  $m$ , we use the fact that

$$m = \frac{y_2 - y_1}{x_2 - x_1} =$$

So the point-slope form for the equation using  $(2, 1)$  as the point is

If we use  $(5, 3)$  instead, then the point-slope form of the equation is

$$y - 3 = \frac{2}{3}(x - 5).$$

These are two different equations for the same line.

## Application: Depreciation.

Linear Depreciation. Office equipment was purchased for \$20,000 and is assumed to have a scrap value of \$2,000 after 10 years. If its value is depreciated linearly (for tax purposes) from \$20,000 to \$2,000:

1. Find the linear equation that relates value ( $V$ ) in dollars to time ( $t$ ) in years. (Hint: you know two points.)

## Application: Depreciation.

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1. Ans:  $V = -1800t + 20000$ . Write a verbal interpretation of the slope of the line.
2. What would be the value of the equipment after 6 years?
3. Graph the equation  $V = -1800t + 20000$  for  $0 \leq t \leq 10$

## Application: Linear Interpolation

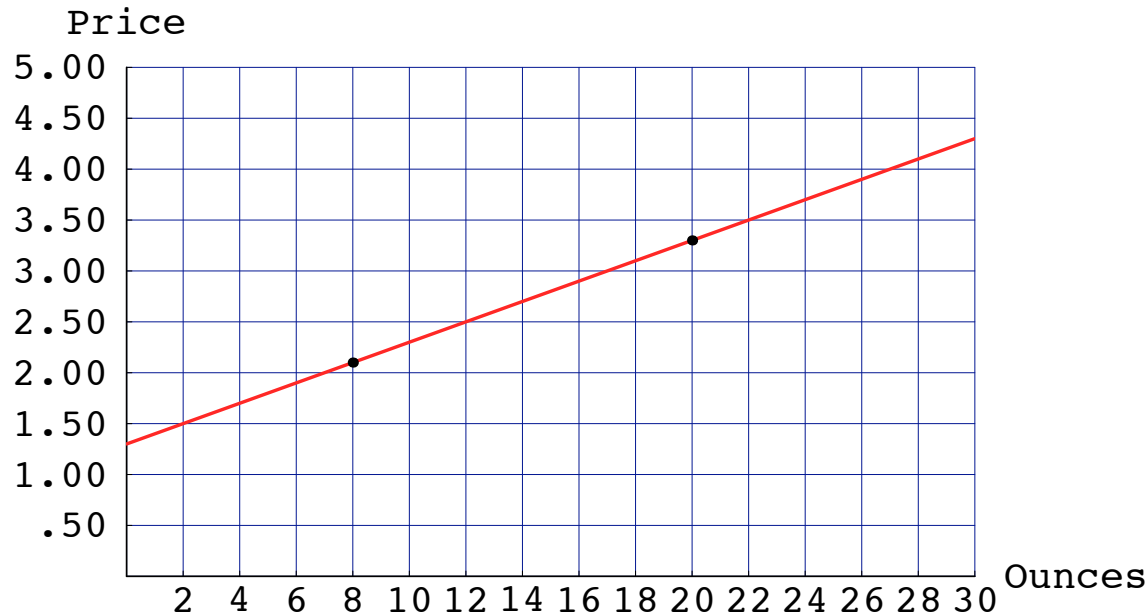
The price of a cup of coffee in a coffee bar depends on the size of the cup. The 8-ounce cup costs \$2.10, but the larger 20-ounce cup costs \$3.30. Without any other information, how could you estimate the cost of a 10-ounce cup, or a 16-ounce cup?

Sometimes business people use a method known as linear interpolation. This means that they **assume** that the price  $p$  of a cup of coffee and the size  $q$  of the cup obey a **linear** equation. So the graph of the linear equation is a line and we know two points on this line:  $(q, p) = (8, 2.10), (20, 3.30)$ .

Units: The price  $p$  of the cup of coffee is given in dollar.  
Size  $q$  of the cup of coffee is given in ounces.

## Application: Linear Interpolation

two points on the line:  $(q, p) = (8, 2.10), (20, 3.30)$ .



Slope: We know that the price of an 8-ounce cup is \$2.10 and the price of a 20-ounce cup is \$3.30. So the change in  $q$  is  $3.30 - 2.10 = \$1.20$  and the change in  $p$  is  $20\text{oz.} - 8\text{oz.} = 12\text{oz.}$ . So the slope is:

$$m = \frac{3.30 - 2.10}{20 - 8} = \frac{1.20}{12} = .10$$

Units for the slope  $m$ : dollars per ounce

## Application: Linear Interpolation

Slope is \$.10 per ounce.

How much does the price increase if the size of the cup increases by 4 ounces?

Point-slope form:  $p - 2.10 = .10(q - 8)$

Slope-intercept form:  $p = .10q + 1.30$ .

$y$ -intercept is \$1.30.

Interpretation: The price of a cup of coffee is \$1.30 plus ten cents an ounce.

## Application: Linear Interpolation

Slope-intercept form:  $p = .10q + 1.30$ .

Use linear interpolation to find the price of a 12-ounce cup of coffee: